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A network DEA approach for series multi-stage processes ☆

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ABSTRACT

We present in this paper a general network DEA approach to deal with efficiency assessments in multi-stage processes. Our approach complies with the composition paradigm, where the efficiencies of the stages are estimated first and the overall efficiency of the system is obtained ex post. We use multi-objective programming as modeling framework. This provides us the means to assess unique and unbiased efficiency scores and, if required, to drive the efficiency assessments effectively in line with specific priorities given to the stages. A direct comparison with the multiplicative decomposition approach on data drawn from the literature brings into light the advantages of our method and some critical points that one should be concerned about when using the multiplicative efficiency decomposition.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for measuring the performance of decision-making units (DMUs) that use multiple inputs to produce multiple outputs. The underlying mathematical model is linear programming. The two basic DEA models, namely the CCR [4] and the BCC [1] models, have become standards in performance measurement under the assumptions of constant and variable returns-to-scale respectively. Conventional DEA deals with one-stage production processes, where the internal structure of the DMUs is not taken into account. The network DEA paradigm, on the other hand, refers to multi-stage processes, where the underlying structure, which indicates the flow of the intermediate measures among the stages, plays a key role in the efficiency assessment. Färe and Grosskopf [11] were among the first to study the efficiency in such processes, represented as network activity analysis models. Castelli et al. [2] provide a comprehensive categorized overview of models and methods developed for different multi-stage production configurations. Kao [15] provides a thorough classification of studies in network DEA, according to the type of the network structure and the model employed. The series and the parallel production processes are two characteristic process configurations studied extensively in the literature. As the latter is beyond the scope of this paper, the reader is referred to [9,12,13–15], where one can identify some links between parallel and series processes. In this paper we focus on multi-stage series production process. The first approaches to deal with the efficiency assessment in two-stage series processes is the *multiplicative decomposition approach* introduced by Kao and Hwang [17] and the *additive decomposition approach* introduced by Chen et al. [5]. Both approaches are based on the reasonable assumption, which ever since is consolidated in the literature, that the weights used for the intermediate measures are the same, no matter if these measures are considered as outputs of the first stage or inputs to the second stage. Liang et al. [20] and Cook et al. [6] studied the efficiency decomposition in two-stage processes using game theoretic concepts. Zhou et al. [22] approached the efficiency decomposition in simple two-stage processes as a Nash bargaining game. Li et al. [19] used a parametric approach to assess the efficiency of DMUs with extra inputs in the second stage, in the frame of the multiplicative approach. Kao et al. [18] used a multi-objective programming approach to the efficiency assessments in network structures. Extensions for multi-stage series processes are given in [2,12,15,16]. Recently, Despotis et al. [8] introduced the *composition paradigm* in two-stage network DEA. Unlike the efficiency decomposition approach, in the composition approach the efficiencies of the two stages are estimated first and the overall efficiency of the DMU is obtained ex post. A major advantage of the assessment method presented in [8] over the additive [5] and the multiplicative [17] methods is that the former provides unique and unbiased efficiency scores for two-stage processes. Its disadvantage, however, is that it cannot be readily extended in multi-stage series processes. This is an effect of the different orientations selected for the first and the second stage, which in fact was made to simplify the models and keep them within the field of linear programming (simplicity at the expense of generality).

In this paper we extend the composition paradigm in general series multi-stage processes, by proposing a multi-objective

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programming approach. Without harming simplicity, our approach overcomes the lack of generality in [8], as long as our model and the solution method proposed can handle any type of series multi-stage process. Our developments makes the direct comparison of the new approach with the multiplicative method [17] possible and fruitful, in a manner that enables us to point out some critical issues that one should take into account when using the multiplicative decomposition method. Unlike the additive and the multiplicative decomposition methods, our new general approach secures the uniqueness of the efficiency scores. Moreover, the efficiency assessments are neutral, in the sense that no implicit priority is assumed for some stages over the others.

The paper is organized as follows. Section 2 is devoted to two-stage processes. We identify four distinct types of processes that cover all possible configurations. In Section 2.1 we unfold our modeling approach in detail with respect to the elementary two-stage process, which assumes that nothing but the external inputs to the first stage enters the system and nothing but the outputs of the second stage leaves the system. A thorough comparison of our method with the multiplicative approach [17] highlights the advantages of the former and points out some critical peculiarities of the latter. In Sections 2.2–2.4, we apply the same approach to other two-stage configurations. When case data are available in the literature, we compare the results obtained by our method with those from other methods. Otherwise, we provide the reader with synthetic data and the corresponding results for testing and validation. In Section 3 we extend our formulations in general multi-stage processes. Conclusions are drawn in Section 4.

2. Two-stage processes

In this section we develop our network DEA approach for the case of two-stage series processes. We follow the composition paradigm introduced in [8]. In the composition paradigm, as opposed to the decomposition approach (cf. [17,5]), the stage efficiencies are estimated without any a priori definition of the overall efficiency of the system. Once the stage efficiencies are estimated, the overall efficiency is computed a posteriori by aggregating the stage efficiencies additively or multiplicatively. We consider four types of processes that cover all possible two-stage series configurations, as depicted in Fig. 1.

Let us introduce the following basic notation:

$j \in J = \{1, \ldots, n\}$: The index set of the n DMUs.
$j_0 \in J$: Denotes the evaluated DMU.
$X_j = (x_{ij}, i = 1, \ldots, m)$: The vector of stage-1 external inputs used by DMU$_j$ (all types).
$Z_j = (z_{pj}, p = 1, \ldots, q)$: The vector of intermediate measures for DMU$_j$ (all types).
$Y_j = \left(y_{rj}, r = 1, \ldots, s\right)$: The vector of stage-2 final outputs produced by DMU$_j$ (all types).
$L_j = (l_{dj}, d = 1, \ldots, a)$: The vector of stage-2 external inputs (types II and IV).
$K_j = (k_{cj}, c = 1, \ldots, b)$: The vector of stage-1 final outputs (types III and IV).
$\eta = (\eta_1, \ldots, \eta_m)$: The vector of weights for the stage-1 external inputs in the fractional model.
$v = (v_1, \ldots, v_m)$: The vector of weights for the stage-1 external inputs in the linear model.
$\varphi = (\varphi_1, \ldots, \varphi_q)$: The vector of weights for the intermediate measures in the fractional model.
$w = (w_1, \ldots, w_q)$: The vector of weights for the intermediate measures in the linear model.
$\omega = (\omega_1, \ldots, \omega_s)$: The vector of weights for the stage-2 outputs in the fractional model.
$u = (u_1, \ldots, u_s)$: The vector of weights for the stage-2 outputs in the linear model.
$\gamma = (\gamma_1, \ldots, \gamma_a)$: The vector of weights for the stage-2 external inputs.
$\mu = (\mu_1, \ldots, \mu_b)$: The vector of weights for the stage-1 final outputs.
e_j^o: The overall efficiency of DMU$_j$.
e_j^1: The efficiency of the first stage for DMU$_j$.
e_j^2: The efficiency of the second stage for DMU$_j$.
E_j^1: The independent efficiency score of the first stage for DMU$_j$.
E_j^2: The independent efficiency score of the first stage for DMU$_j$.

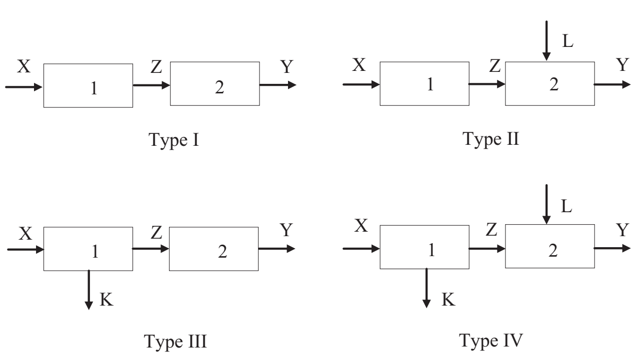

Fig. 1. The four types of series two-stage processes.

2.1. Type I structure

Consider the elementary case (Type I) where each DMU transforms some external inputs X to final outputs Y via the intermediate measures Z with a two-stage process, as depicted in Fig. 1. In this basic setting, nothing but the external inputs to the first stage enters the system and nothing but the outputs of the second stage leaves the system. Typically, the efficiency of the first and the second stage of a DMU j are defined as follows:

$$e_j^1 = \frac{\varphi Z_j}{\eta X_j}, \quad e_j^2 = \frac{\omega Y_j}{\varphi Z_j}$$

The overall efficiency of DMUj is defined as the ratio of the total virtual exogenous output to the total virtual exogenous input:

$$e_j^o = \frac{\omega Y_j}{\eta X_j}$$

Consider the basic input oriented CRS-DEA models that estimate the stage-1 and the stage-2 efficiency for the evaluated unit j_0 independently:

$$\begin{aligned} E_{j_0}^1 = \max \ & \frac{\varphi Z_{j_0}}{\eta X_{j_0}} \\ \text{s.t.} \ & \\ & \varphi Z_j - \eta X_j \le 0, \ j = 1, \ldots, n \\ & \eta \ge \varepsilon, \ \varphi \ge \varepsilon \end{aligned} \tag{1}$$

$$\begin{aligned} E_{j_0}^2 = \max \ & \frac{\omega Y_{j_0}}{\varphi Z_{j_0}} \\ \text{s.t.} \ & \\ & \omega Y_j - \varphi Z_j \le 0, \ j = 1, \ldots, n \\ & \varphi \ge \varepsilon, \ \omega \ge \varepsilon \end{aligned} \tag{2}$$

In order to link the efficiency assessments of the two stages, it is universally accepted that the weights associated with the intermediate measures are the same, no matter if these measures are considered as outputs of the first stage or inputs to the second stage. Appending the constraints of model (1) to model (2) and vice versa we get the following augmented models (3) and (4) for the first and the second stage respectively:

$$\begin{aligned} E^1_{j_0} = \ & \max \frac{\varphi Z_{j_0}}{\eta X_{j_0}} \\ & \text{s.t.} \\ & \varphi Z_j - \eta X_j \le 0, \ j=1,\dots,n \\ & \omega Y_j - \varphi Z_j \le 0, \ j=1,\dots,n \\ & \eta \ge \varepsilon, \ \varphi \ge \varepsilon, \ \omega \ge \varepsilon \end{aligned} \tag{3}$$

$$\begin{aligned} E^2_{j_0} = \ & \max \frac{\omega Y_{j_0}}{\varphi Z_{j_0}} \\ & \text{s.t.} \\ & \varphi Z_j - \eta X_j \le 0, \ j=1,\dots,n \\ & \omega Y_j - \varphi Z_j \le 0, \ j=1,\dots,n \\ & \eta \ge \varepsilon, \ \varphi \ge \varepsilon, \ \omega \ge \varepsilon \end{aligned} \tag{4}$$

As noticed in [8], the optimal solutions of (1) and (2) are also optimal in (3) and (4) respectively. Models (3) and (4) have common constraints and, thus, they form the following bi-objective program:

$$\begin{aligned} & \max \frac{\varphi Z_{j_0}}{\eta X_{j_0}} \\ & \max \frac{\omega Y_{j_0}}{\varphi Z_{j_0}} \\ & \text{s.t.} \\ & \varphi Z_j - \eta X_j \le 0, \ j=1,\dots,n \\ & \omega Y_j - \varphi Z_j \le 0, \ j=1,\dots,n \\ & \eta \ge \varepsilon, \ \varphi \ge \varepsilon, \ \omega \ge \varepsilon \end{aligned} \tag{5}$$

Applying the Charnes–Cooper [3] transformation (C–C transformation hereafter) with respect to the first objective function, i.e. multiplying all the terms of the fractional objective functions and the constraints by $t>0$, such that $t\eta X_{j_0}=1$ and setting $t\eta = v$, $t\omega = u$, $t\varphi = w$ we get the following equivalent bi-objective program, whose second objective function is still fractional:

$$\begin{aligned} & \max wZ_{j_0} \\ & \max \frac{uY_{j_0}}{wZ_{j_0}} \\ & \text{s.t.} \\ & vX_{j_0} = 1 \\ & wZ_j - vX_j \le 0, \ j=1,\dots,n \\ & uY_j - wZ_j \le 0, \ j=1,\dots,n \\ & v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon \end{aligned} \tag{6}$$

Solving the linear equivalents of models (3) and (4) one gets the independent efficiency scores $E^1_{j_0}$ and $E^2_{j_0}$ of the two stages respectively. In terms of multi-objective programming (MOP), the vector $(E^1_{j_0}, E^2_{j_0})$ constitutes the ideal point of the bi-objective program (6) in the objective functions space. The efficiencies of the two stages can be obtained by solving the bi-objective program (6). However, as the ideal point is not generally attainable, solving a MOP means finding efficient (Pareto optimal) solutions in the variable space that are mapped on the Pareto front in the objective functions space, i.e. solutions that they cannot be altered to increase the value of one objective function without decreasing the value of at least one other objective function. The model (7) below employs the weighted Tchebycheff norm (L_∞ norm) to locate a point on the Pareto front, by minimizing the maximum of the weighted deviations $t_1(E^1_{j_0} - e^1_{j_0})$ and $t_2\left(E^2_{j_0} - e^2_{j_0}\right)$ of $(e^1_{j_0} = wZ_{j_0}, \ e^2_{j_0} = uY_{j_0}/wZ_{j_0})$ from the ideal point $(E^1_{j_0}, E^2_{j_0})$, with weights $t_1>0$ and $t_2>0$.

$$\begin{aligned} & \min \delta \\ & \text{s.t.} \\ & t_1(E^1_{j_0} - wZ_{j_0}) \le \delta \\ & t_2\left(E^2_{j_0} - \frac{uY_{j_0}}{wZ_{j_0}}\right) \le \delta \\ & vX_{j_0} = 1 \\ & wZ_j - vX_j \le 0, \ j=1,\dots,n \\ & uY_j - wZ_j \le 0, \ j=1,\dots,n \\ & v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \delta \ge 0 \end{aligned} \tag{7}$$

Every optimal solution of (7) is *weakly* efficient (*weakly* Pareto optimal) solution for (6) (see, e.g., [10]). At optimality, at least one of the first two constraints in (7) will be binding. Assuming that there is no stated preference information that gives priority to one of the two stages, we employ in our assessments the unweighted Tchebycheff norm, i.e. we assume $t_1 = t_2 = 1$, and we get the following:

$$\begin{aligned} & \min \delta \\ & \text{s.t.} \\ & E^1_{j_0} - wZ_{j_0} \le \delta \\ & \left(E^2_{j_0} - \delta\right) wZ_{j_0} - uY_{j_0} \le 0 \\ & vX_{j_0} = 1 \\ & wZ_j - vX_j \le 0, \ j=1,\dots,n \\ & uY_j - wZ_j \le 0, \ j=1,\dots,n \\ & v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \delta \ge 0 \end{aligned} \tag{8}$$

Although model (8) is non-linear, it can be easily solved by bisection search (cf. [7]). Clearly, $0 \le \delta \le 1$. Hence bisection search can be performed in the bounded interval [0,1] as follows. Let $\underline{\delta}$ be a lower bound of δ for which the constraints of (8) are not consistent (initially $\underline{\delta}=0$) and $\overline{\delta}$ an upper bound of δ for which the constraints are consistent (initially $\overline{\delta}=1$). Then the consistency of the constraints is tested for $\delta' = (\underline{\delta} + \overline{\delta})/2$. If they are consistent, δ' will replace $\overline{\delta}$; if they are not it will replace $\underline{\delta}$. The bisection continues until both bounds come sufficiently close to each other. Let $(\delta^*, v^*, w^*, u^*)$ be an optimal solution of (8) and

$$e^{1*}_{j_0} = \frac{w^* Z_{j_0}}{v^* X_{j_0}} = w^* Z_{j_0}, \quad e^{2*}_{j_0} = \frac{u^* Y_{j_0}}{w^* Z_{j_0}}$$

The model (9) below provides a Pareto optimal solution to (6). The model (9) is equivalent to employing lexicographically (in a second phase) the L_1 norm on the set of optimal solutions of (8) (see, e.g. [21]).

$$\begin{aligned} & \max s_1 + s_2 \\ & \text{s.t.} \\ & E^1_{j_0} - wZ_{j_0} + s_1 = \delta^* \\ & \left(E^2_{j_0} - \delta^*\right) wZ_{j_0} - uY_{j_0} + s_2 w^* Z_{j_0} = 0 \\ & vX_{j_0} = 1 \\ & wZ_j - vX_j \le 0, \ j=1,\dots,n \\ & uY_j - wZ_j \le 0, \ j=1,\dots,n \\ & v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon \\ & \delta^* \ge s_1 \ge 0, \ \delta^* \ge s_2 \ge 0 \end{aligned} \tag{9}$$

In (9), δ^* is the optimal value of the objective function of (8) and $w^* Z_{j_0}$ the optimal virtual intermediate measure derived by model (8). Notice here that the term $w^* Z_{j_0}$ is used as an effective substitute of wZ_{j_0} to secure the linearity of the model. In case that $s_2 > 0$ in the optimal solution of (9), the program is solved iteratively by replacing in each iteration the weights w in the coefficient of s_2 with the optimal weights w obtained in the preceding iteration, until the stage efficiencies in two successive iterations remain unchanged (cf. [7] for a similar treatment). The same holds for the second phase programs in types II–IV as well as in the general case presented in the next

Table 1
Results obtained from the additive and the multiplicative decomposition methods.

DMU	Chen et al. [5]			Kao and Hwang [17]		
	e^1	e^2	e^o	e^1	e^2	e^o
1	0.9926	0.7045	0.8491	0.9926	0.7045	0.6992
2	0.9985	0.6257	0.8122	0.9985	0.6257	0.6248
3	0.6900	1	0.8166	0.6900	1	0.6900
4	0.7243	0.4200	0.5965	0.7243	0.4200	0.3042
5	0.8307	0.9233	0.8727	0.8307	0.9233	0.7670
6	0.9606	0.4057	0.6887	0.9606	0.4057	0.3897
7	0.7521	0.3522	0.5804	0.6706	0.4124	0.2766
8	0.7256	0.3780	0.5795	0.6630	0.4150	0.2752
9	1	0.2233	0.6116	1	0.2233	0.2233
10	0.8615	0.5408	0.7131	0.8615	0.5408	0.4660
11	0.7291	0.2068	0.5088	0.6468	0.2534	0.1639
12	1	0.7596	0.8798	1	0.7596	0.7596
13	0.8107	0.2431	0.5565	0.6720	0.3093	0.2078
14	0.7246	0.3740	0.5773	0.6699	0.4309	0.2886
15	1	0.6138	0.8069	1	0.6138	0.6138
16	0.8856	0.3615	0.6395	0.8856	0.3615	0.3202
17	0.7232	0.4597	0.6126	0.6276	0.5736	0.3600
18	0.7935	0.3262	0.5868	0.7935	0.3262	0.2588
19	1	0.4112	0.7056	1	0.4112	0.4112
20	0.9332	0.5857	0.7654	0.9332	0.5857	0.5465
21	0.7505	0.2623	0.5412	0.7321	0.2743	0.2008
22	0.5895	1	0.7418	0.5895	1	0.5895
23	0.8426	0.4989	0.6854	0.8426	0.4989	0.4203
24	1	0.0870	0.5435	0.4287	0.3145	0.1348

sections. The optimal solution $(\hat{v}, \hat{w}, \hat{u})$ of (9) is a Pareto optimal solution of (6) and the efficiency scores for unit j_0 in the first and the second stage as well as the overall efficiency of the system are respectively:

$$\hat{e}^1_{j_0} = \frac{\hat{w}Z_{j_0}}{\hat{v}X_{j_0}} = \hat{w}Z_{j_0}, \quad \hat{e}^2_{j_0} = \frac{\hat{u}Y_{j_0}}{\hat{w}Z_{j_0}}, \quad \hat{e}^o_{j_0} = \frac{\hat{u}Y_{j_0}}{\hat{v}X_{j_0}} = \hat{u}Y_{j_0}$$

with $\hat{e}^o_{j_0} = \hat{e}^1_{j_0} \cdot \hat{e}^2_{j_0}$. Since the optimal solution of (8) is weakly Pareto optimal, in (9), at most one of the two optimal values of the variables $\hat{s}_1$ and $\hat{s}_2$ will be strictly positive. If $\hat{s}_1 = 0$ and $\hat{s}_2 = 0$, then the optimal solution of (8) is Pareto optimal.

2.1.1. Illustration

For comparison purposes, we apply models (8) and (9) to the data originally presented in [17] and used in many other studies. The case concerns the performance measurement of 24 Taiwanese non-life insurance companies. The authors considered a two-stage production process with two inputs (Operation expenses—$X1$ and Insurance expenses—$X2$), two intermediate measures (Direct written premiums—$Z1$ and Reinsurance premiums—$Z2$) and two final outputs (Underwriting profit—$Y1$ and Investment profit—$Y2$). For the complete data set the reader is referred to the original article [17]. Table 1 summarizes the results obtained by applying the *additive decomposition* method [5] (columns 2–4) and the *multiplicative decomposition* method [17] (columns 5–7).

Table 2 exhibits the results obtained by applying the proposed approach. Specifically, columns 2 and 3 present the independent (ideal) efficiency scores for stage-1 and stage-2 respectively, columns 4 and 5 present the stage-1 and stage-2 efficiency scores, whereas the last column presents the overall efficiency scores. Notice here that in all cases (DMUs), the model (8) provided Pareto optimal solutions, i.e. model (9) did not alter the efficiency scores obtained from (8).

2.1.2. Comparison of the new approach with the multiplicative decomposition approach

Recently, Despotis et al. [8] showed that the additive decomposition approach of Chen et al. [5] biases the efficiency assessments in favor of the second stage. In this section we will show the relation of our approach with the multiplicative decomposition method of Kao and Hwang [17]. Recall here that the multiplicative decomposition model assumes that the overall efficiency is the product of the stage efficiencies:

Table 2
Results obtained from model (9) (same as from model (8)).

DMU	E^1	E^2	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^o$
1	0.9926	0.7134	0.9847	0.7055	0.6946
2	0.9985	0.6275	0.9971	0.6260	0.6242
3	0.6900	1	0.6900	1	0.6900
4	0.7243	0.4323	0.7125	0.4205	0.2996
5	0.8375	1	0.7912	0.9537	0.7545
6	0.9637	0.4057	0.9618	0.4038	0.3884
7	0.7521	0.5378	0.6385	0.4243	0.2709
8	0.7256	0.5113	0.6375	0.4232	0.2698
9	1	0.2920	0.9408	0.2328	0.2190
10	0.8615	0.6736	0.7557	0.5678	0.4290
11	0.7405	0.3267	0.6594	0.2455	0.1619
12	1	0.7596	1	0.7596	0.7596
13	0.8107	0.5435	0.6075	0.3404	0.2068
14	0.7246	0.5178	0.6463	0.4395	0.2840
15	1	0.7047	0.9341	0.6389	0.5968
16	0.9072	0.3847	0.8843	0.3618	0.3199
17	0.7233	1	0.4419	0.7186	0.3175
18	0.7935	0.3737	0.7572	0.3373	0.2554
19	1	0.4158	0.9962	0.4120	0.4104
20	0.9332	0.9014	0.7289	0.6970	0.5081
21	0.7505	0.2795	0.7400	0.2690	0.1991
22	0.5895	1	0.5895	1	0.5895
23	0.8501	0.5599	0.8020	0.5119	0.4106
24	1	0.3351	0.7978	0.1328	0.1060

$$e^1_j = \frac{wZ_j}{vX_j}, \quad e^2_j = \frac{uY_j}{wZ_j}, \quad e^o_j = e^1_j \cdot e^2_j = \frac{uY_j}{vX_j}$$

The model below estimates the stage efficiencies by optimizing the overall efficiency:

$$\begin{aligned} e^o_{j_0} = {} & \max uY_{j_0} \\ & \text{s.t.} \\ & vX_{j_0} = 1 \\ & wZ_j - vX_j \le 0, \ j = 1, \dots, n \\ & uY_j - wZ_j \le 0, \ j = 1, \dots, n \\ & v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon \end{aligned} \tag{10}$$

Once an optimal solution (v^*, w^*, u^*) of model (10) is obtained, the overall efficiency and the stage efficiencies are calculated as follows:

$$e^o_{j_0} = u^*Y_{j_0}, \quad e^1_{j_0} = \frac{w^*Z_{j_0}}{v^*X_{j_0}} = w^*Z_{j_0}, \quad e^2_{j_0} = \frac{u^*Y_{j_0}}{w^*Z_{j_0}} = \frac{e^o_{j_0}}{e^1_{j_0}}$$

The difference between our method and the multiplicative decomposition method is conceptual rather than structural. In fact, our method comes under the composition paradigm introduced in [8]. Structurally, models (6) and (10) have exactly the same constraints and differ only in the objective functions. That is both models have the same feasible region. Model (6) is a bi-objective program (vector-maximization model) with the objectives representing the stage-1 and stage-2 efficiencies. The overall efficiency of the system is obtained by the Pareto optimal solution of (6) that locates the stage efficiencies as close as possible to their ideal values in the minmax sense. In model (10), on the other hand, the overall efficiency of the system is maximized and the stage efficiencies are obtained as offspring by decomposing the overall efficiency. The structural similarity of models (6) and (10) enables plotting their objective functions space jointly. Fig. 2 below is a general representation of the objective functions space of models (6) and (10) for an evaluated unit (X_0, Z_0, Y_0). Actually, it

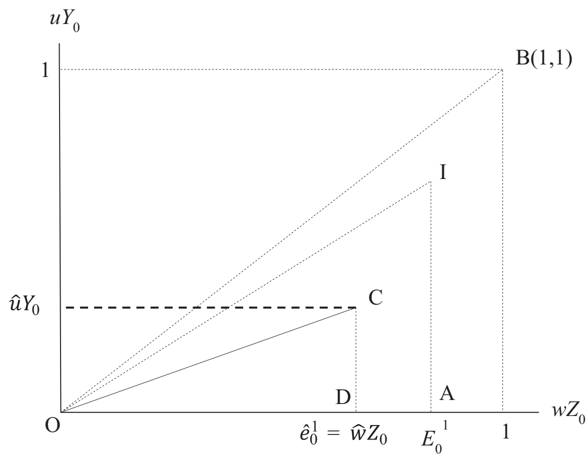

Fig. 2. General representation of the objective functions space of models (6) and (10).

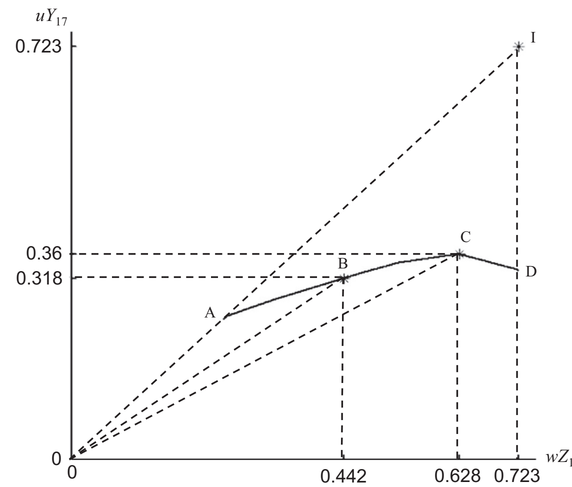

Fig. 3. The Pareto front of DMU 17.

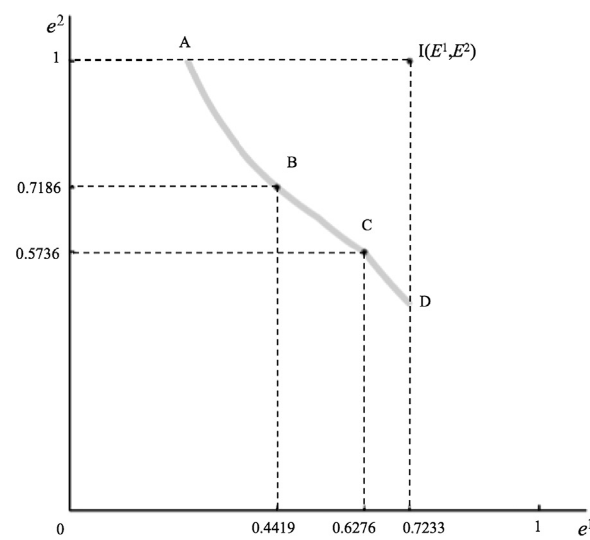

Fig. 4. The conventional Pareto front for DMU 17 in the (e^1, e^2) space.

is the plane in the three-dimensional space (vX_0, wZ_0, uY_0) that is vertical to the axis vX at $vX_0=1$. The horizontal axis represents for both models the stage-1 efficiency. The vertical axis represents the overall efficiency as per model (10), i.e. the product of stage-1 and stage-2 efficiencies for both models.

The point B(1,1) represents the boundaries of the objective functions values and corresponds to an overall efficient unit with $e_{j_0}^1 = w^*Z_{j_0} = 1$ and $e_{j_0}^o = u^*Y_{j_0} = 1$. Then, the efficiency of stage-2 is $e_{j_0}^2 = u^*Y_{j_0}/w^*Z_{j_0} = 1$ and is represented by the slope of the bisecting line OB. The point I corresponds to the stage-1 and stage-2 ideal (independent) efficiency scores of the evaluated unit and is formed as the intersection of the vertical line to the horizontal axis at E_0^1 and a line form the origin with slope E_0^2, i.e. $E_0^2 = tan\, I\hat{O}A$. The point C is located by the model (9) on the Pareto front of model (6) and is formed as the intersection of the vertical line to the horizontal axis at $\hat{e}_0^1 = \hat{w}Z_0$ and the line from the origin with slope $\hat{e}_0^2 = \hat{u}Y_0/\hat{w}Z_0$. The abscissa of C is the stage-1 efficiency, whereas its ordinate is the overall efficiency of the evaluated unit as defined in the multiplicative model. Thus, C represents the Pareto front point derived by the multiplicative model (10) if and only if its ordinate is maximal.

Consider now the parametric version of model (8) that is solved for different values of the parameters $t_1 > 0$ and $t_2 > 0$, such that $t_1 + t_2 = 1$.

$$
\begin{aligned}
&\min \delta \\
&\quad \text{s.t.} \\
&\quad t_1 wZ_{j_0} + \delta \geq t_1 E_{j_0}^1 \\
&\quad t_2 uY_{j_0} - (t_2 E_{j_0}^2 - \delta) wZ_{j_0} \geq 0 \\
&\quad vX_{j_0} = 1 \\
&\quad wZ_j - vX_j \leq 0,\ j = 1, \dots, n \\
&\quad uY_j - wZ_j \leq 0,\ j = 1, \dots, n \\
&\quad v \geq \varepsilon,\ w \geq \varepsilon,\ u \geq \varepsilon,\ \delta \geq 0
\end{aligned} \tag{11}
$$

For every t_1 and t_2, model (11) locates a point on the Pareto front of model (6). That is, model (11) can be used as an instrument to generate the Pareto front of model (6). The greatest is the value of t_1 than t_2 the highest is the priority given to stage-1 over stage-2 and vice versa. The crooked line ABCD in Fig. 3 represents the Pareto front of model (6) for DMU 17 (cf. Tables 1 and 2). Point I depicts the ideal (independent) stage efficiencies of this unit. Particularly, its abscissa is $E_{17}^1 = 0.7233$ and the slope of the line OI is $E_{17}^2 = 1$. Point C is the point on the Pareto front that corresponds to the solution obtained by the multiplicative model of Kao and Hwang [17]. Its ordinate is $e_{17}^o = 0.36$, which is maximal, its abscissa is $e_{17}^1 = 0.6276$ whereas the stage-2 efficiency is $e_{17}^2 = e_{17}^o / e_{17}^1 = 0.5736$ and is represented by the slope of the line OC. Point B depicts the point on the Pareto front obtained by our model (9). The abscissa of point B is the stage-1 efficiency $\hat{e}_{17}^1 = 0.4419$, the slope of the line OB is the stage-2 efficiency $\hat{e}_{17}^2 = 0.7186$, whereas the ordinate of point B is the overall efficiency $\hat{e}_{17}^o = 0.3175$. It is clear that generally holds $\hat{e}_j^o \leq e_j^o$ because e_j^o is maximal.

Fig. 4 exhibits the conventional Pareto front for DMU 17 in the objective functions space of (6) with the horizontal and the vertical axes representing respectively the stage-1 and the stage-2 efficiency scores. Point I is formed by the ideal efficiency scores ($E_{17}^1 = 0.7233$, $E_{17}^2 = 1$). The curve ABCD is the Pareto front for unit 17, the point B(0.4419, 0.7186) is the Pareto optimal solution obtained by model (8) and is uniquely formed by the intersection of the Pareto front with a ray from the ideal point I with direction $(-1, -1)$. Point C(0.6276, 0.5736) represents the solution obtained by the multiplicative model (10).

The model (9) locates a unique point on the Pareto front, i.e. it estimates unique efficiency scores for the two stages. Given that the unweighted Tchebycheff norm is employed in (8), no priority is assumed for one stage over the other. If, however, one is to assign different priorities to the two stages, the efficiency assessment can be performed via the weighted variant (11), with specific values for the parameters t_1 and t_2 that reflect the analyst's preference.

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Table 3
Synthetic data and results obtained by model (10) and post-optimality analysis.

DMU	X1	X2	Z1	Z2	Y1	Y2	e^1	e^2	e^o	e^1_{max}	e^1_-	e^2_{max}	e^2_-	DMU
1	69.5	68.6	56.6	84.4	48.7	62.8	0.2316	0.5070	0.1174	0.2316	0.2316	0.5070	0.5070	1
2	40.2	66.2	88	47.2	85.8	28.3	0.3265	0.7508	0.2451	0.3265	0.3265	0.7508	0.7508	2
3	81.3	89.8	44.4	18.4	38.3	20.7	0.0866	0.6844	0.0593	0.0866	0.0866	0.6844	0.6844	3
4	55	97.9	28.7	41.6	38.2	10.3	0.1344	0.5830	0.0784	0.1344	0.1344	0.5830	0.5830	4
5	56.2	59.1	26.5	52.7	44.2	17.4	0.1688	0.5823	0.0983	0.1688	0.1688	0.5823	0.5823	5
6	64.8	64.4	14.7	70.5	86.6	22.9	0.1685	1	0.1685	0.1685	0.1685	1	1	6
7	79.2	68.1	63.5	39.3	47.6	35	0.1644	0.5613	0.0923	0.1644	0.1644	0.5613	0.5613	7
8	36	74.3	66.6	57.4	40.3	94.8	0.4297	0.6953	0.2987	0.4297	0.4145	0.7208	0.6953	8
9	10.8	10.3	46.5	47.9	57.5	95.2	1	1	1	1	1	1	1	9
10	17.7	93.6	35.9	58.7	45.9	12	0.5235	0.5149	0.2696	0.5235	0.5235	0.5149	0.5149	10
11	38.8	97.5	55.2	41.7	60.5	82.7	0.3113	0.8189	0.2550	0.3113	0.3113	0.8189	0.8189	11
12	60.9	96.4	86	28.9	93.1	72.3	0.2006	1	0.2006	0.2006	0.2006	1	1	12
13	70.3	45.8	65.3	35.3	34.3	98.8	0.3158	0.7390	0.2334	0.3158	0.2334	1	0.7390	13
14	20.5	75.6	13.1	60	53.3	18.3	0.3759	0.7190	0.2703	0.3759	0.3759	0.7190	0.7190	14
15	17.9	74.8	54.2	66.7	52.1	15.8	0.6443	0.4696	0.3026	0.6443	0.6443	0.4696	0.4696	15
16	51.8	19.8	52.3	74.2	73.6	84.7	0.6980	0.8163	0.5698	0.6980	0.6980	0.8163	0.8163	16
17	11.3	27.3	42.7	72.3	68.9	37.4	1	0.6694	0.6694	1	1	0.6694	0.6694	17
18	58.7	42.1	95.9	26.6	51.6	96.4	0.5046	0.4910	0.2477	0.5046	0.3021	0.8201	0.4910	18
19	41.4	51.6	83	75.4	20.5	72	0.4656	0.4237	0.1973	0.4656	0.4537	0.4348	0.4237	19
20	99.7	87.1	87.5	96.9	58.6	39	0.2311	0.3739	0.0864	0.2311	0.2311	0.3739	0.3739	20
21	25.6	14.6	52	19.1	44.3	51.3	0.5293	0.8807	0.4662	0.5293	0.5293	0.8807	0.8807	21
22	65.1	97.3	79.4	68	53.8	55.5	0.2438	0.4927	0.1201	0.2438	0.2438	0.4927	0.4927	22
23	40.4	33	74.5	21.7	13.9	55.7	0.5001	0.3652	0.1826	0.5001	0.3031	0.6025	0.3652	23
24	19.4	20.1	77.5	74.1	60.9	71	0.8855	0.5673	0.5023	0.8855	0.8855	0.5673	0.5673	24
25	54.2	99.3	20.8	69.9	47.8	12.2	0.1867	0.5291	0.0988	0.1867	0.1867	0.5291	0.5291	25
26	80.1	27.5	51.3	95.8	21.7	12.6	0.5850	0.1674	0.0979	0.5850	0.5850	0.1674	0.1674	26
27	82.9	38.1	43.3	75.3	16.8	26.6	0.3403	0.2273	0.0774	0.3403	0.3403	0.2273	0.2273	27
28	98.6	81.8	93.8	15.9	40.3	35.8	0.1455	0.4735	0.0689	0.1455	0.1455	0.4735	0.4735	28
29	77.3	40.3	95.6	52.5	96.1	44.2	0.3766	0.7660	0.2885	0.3766	0.3766	0.7660	0.7660	29
30	38.6	58.3	37.8	66.1	16	69.9	0.2274	0.9032	0.2054	0.2618	0.2274	0.9032	0.7847	30

Each distinct pair (t_1, t_2) locates a point on the Pareto front. Since the model (11) can locate any point on the Pareto front, it can locate point C in Fig. 3 (point C in Fig. 4) as well. Indeed, solving model (11) for $t_1 = 0.81668$, $t_2 = 0.18332$ we get the same stage and overall efficiencies as those obtained by the multiplicative method. Notice however, that in this case the stage-1 is over-weighted significantly at the expense of the stage-2. This is an indication that the multiplicative decomposition method, when maximizing the overall efficiency of a unit, may implicitly, yet unreasonably, assume different and, interestingly, DMU-specific priorities for the two stages. Thus, the decomposition of the overall efficiency to the stage efficiencies may bias the efficiency assessments in favor of one stage over the other and it does not provide the analyst with the necessary information to communicate the results, as concerns the priorities of the stages.

Kao and Hwang [17] proposed a pair of post-optimality models to check the uniqueness of the efficiency decomposition. As shown in Fig. 3, the efficiency decomposition for DMU 17 is unique at point C. Although this holds for all the 24 units in Table 1, it is not a general property of the multiplicative decomposition in model (10). Table 3 presents a synthetic case of 30 DMUs with two inputs ($X1$, $X2$), two intermediate measures ($Z1$, $Z2$) and two outputs ($Y1$,$Y2$) drawn form a uniform distribution in the interval [10,100]. Columns 8–10 present the overall and stage efficiency scores obtained by the multiplicative decomposition model (10). Columns 11–14 present alternative efficiency decompositions that maintain the optimal overall efficiency score e^o. They are calculated by applying the post-optimality check proposed in [17]. Specifically, columns 11 and 12 provide the maximal and the minimal efficiencies for stage-1 that maintain the overall efficiency score. Respectively, the maximal and the minimal efficiencies for stage-2 are given in columns 13 and 14. These results show that the efficiency decomposition for the units 8, 13, 18, 19, 23 and 30 is not unique.

The crooked line ABD in Fig. 5 depicts the Pareto front generated by model (11) for unit 18. Notice again that applying model (8) to the data of Table 3 generates Pareto optimal solutions for all the units, i.e. the second-phase model (9) does not alter the efficiency scores obtained by the former.

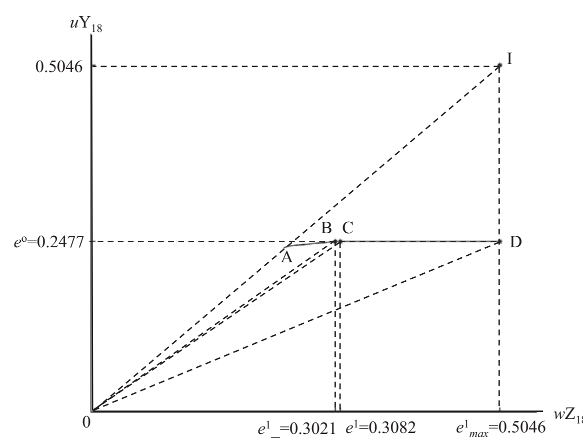

Fig. 5. Non-unique efficiency decomposition of unit 18.

The point I depicts the ideal solution of (6) ($E^1_{18} = 0.5046$, $E^2_{18} = 1$). Actually, the independent (ideal) efficiency score of stage-2 is represented by the slope of the line OI. The segment BD of the Pareto front is parallel to the horizontal axis and all the points on it correspond to equivalent efficiency decompositions that maintain the same overall efficiency $e^o_{18} = 0.2477$. Points B and D depict the two extreme decompositions ($e^1_- = e^o/e^2_{max} = 0.3021$, $e^2_{max} = 0.8201$) and ($e^1_{max} = 0.5046$, $e^2_- = e^o/e^1_{max} = 0.4910$) respectively. The slopes of the lines OB and OD represent the stage-2 efficiency scores e^2_{max} and e^2_- respectively. Point C represents the unique Pareto optimal point obtained by our model (8) with $\hat{e}^1 = 0.3082$, $\hat{e}^2 = 0.8037$ and $\hat{e}^o = 0.2477$. Fig. 6 exhibits the conventional form of the Pareto front for unit 18. The counterpart in Fig. 6 of the segment BD of the Pareto front in Fig. 5 is the

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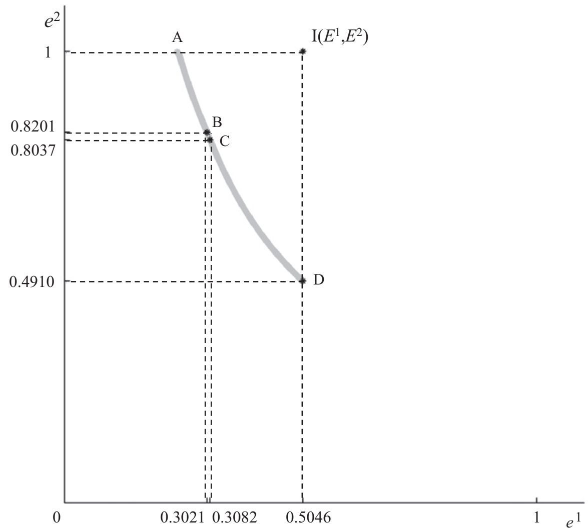

Fig. 6. The conventional Pareto front for unit 18 in the (e^1, e^2) space.

curve BD, which, in fact, consists of an infinite number of alternative efficiency decompositions of the overall efficiency $e^o = 0.2477$. Contrariwise, model (8) generates the unique pair of Pareto optimal efficiency scores depicted on point C. Summarizing, unlike the Kao and Hwang's [17] multiplicative efficiency decomposition method, our approach generates unique and unbiased efficiency scores.

2.2. Type II structure

In the structure of type II, the second stage uses some extra external inputs L beyond the intermediate measures as depicted in Fig. 1. In this case, the efficiency of the first and the second stage of DMU j are defined as follows:

$$e_j^1 = \frac{wZ_j}{vX_j}, \quad e_j^2 = \frac{uY_j}{wZ_j + \gamma L_j}$$

The overall efficiency of DMUj is defined as the ratio of the total virtual exogenous output to the total virtual exogenous input:

$$e_j^o = \frac{uY_j}{vX_j + \gamma L_j}$$

Similarly to Type I, the bi-objective program for estimating the efficiencies of the two stages is as follows:

$$\begin{aligned} &\max \frac{wZ_{j_0}}{vX_{j_0}} \\ &\quad \max \frac{uY_{j_0}}{wZ_{j_0} + \gamma L_{j_0}} \\ &\quad \text{s.t.} \\ &\quad wZ_j - vX_j \le 0, \ j = 1, \dots, n \\ &\quad uY_j - wZ_j - \gamma L_j \le 0, \ j = 1, \dots, n \\ &\quad v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \gamma \ge \varepsilon \end{aligned} \tag{12}$$

Applying the C–C transformation to (12) on the basis of the denominator of the first objective function, we get the following:

$$\begin{aligned} &\max wZ_{j_0} \\ &\quad \max \frac{uY_{j_0}}{wZ_{j_0} + \gamma L_{j_0}} \\ &\quad \text{s.t.} \\ &\quad vX_{j_0} = 1 \\ &\quad wZ_j - vX_j \le 0, \ j = 1, \dots, n \\ &\quad uY_j - wZ_j - \gamma L_j \le 0, \ j = 1, \dots, n \\ &\quad v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \gamma \ge \varepsilon \end{aligned} \tag{13}$$

Notice here that there is a variable transformation from (12) to (13) (see previous section) but we use the same variable names for the economy of notation. The same simplification is adopted in the next sections.

The minmax model that calculates the stage-1 and stage-2 efficiency scores at a minimum distance (unweighted L_∞ norm) from their ideal counterparts is as follows:

$$\begin{aligned} &\min \delta \\ &\quad \text{s.t.} \\ &\quad E_{j_0}^1 - wZ_{j_0} \le \delta \\ &\quad (E_{j_0}^2 - \delta)(wZ_{j_0} + \gamma L_{j_0}) - uY_{j_0} \le 0 \\ &\quad vX_{j_0} = 1 \\ &\quad wZ_j - vX_j \le 0, \ j = 1, \dots, n \\ &\quad uY_j - wZ_j - \gamma L_j \le 0, \ j = 1, \dots, n \\ &\quad v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \gamma \ge \varepsilon, \ \delta \ge 0 \end{aligned} \tag{14}$$

The ideal efficiency scores are obtained by considering (12) with one objective function at a time and solving its linear equivalent derived by the C–C transformation. The optimal solution of (14) is weakly Pareto optimal solution of (13). As explained in the previous section, model (14) can be solved by bisection search. Let $(\delta^*, v^*, w^*, u^*, \gamma^*)$ be an optimal solution of (14) and

$$e_{j_0}^{1*} = \frac{w^* Z_{j_0}}{v^* X_{j_0}} = w^* Z_{j_0}, \quad e_{j_0}^{2*} = \frac{u^* Y_{j_0}}{w^* Z_{j_0} + \gamma^* L_{j_0}}$$

The second phase program (15) below provides a Pareto optimal solution to (13):

$$\begin{aligned} &\max s_1 + s_2 \\ &\quad \text{s.t.} \\ &\quad E_{j_0}^1 - wZ_{j_0} + s_1 = \delta^* \\ &\quad (E_{j_0}^2 - \delta^*)(wZ_{j_0} + \gamma L_{j_0}) - uY_{j_0} + s_2(w^* Z_{j_0} + \gamma^* L_{j_0}) = 0 \\ &\quad vX_{j_0} = 1 \\ &\quad wZ_j - vX_j \le 0, \ j = 1, \dots, n \\ &\quad uY_j - wZ_j - \gamma L_j \le 0, \ j = 1, \dots, n \\ &\quad v \ge \varepsilon, \ w \ge \varepsilon, \ u \ge \varepsilon, \ \gamma \ge \varepsilon \\ &\quad \delta^* \ge s_1 \ge 0, \ \delta^* \ge s_2 \ge 0 \end{aligned} \tag{15}$$

Given the optimal solution $(\hat{s}_1, \hat{s}_2, \hat{v}, \hat{w}, \hat{u}, \hat{\gamma})$ of (15), the efficiency scores for unit j_0 in the first and the second stage as well as the overall efficiency of the system are respectively:

$$\hat{e}_{j_0}^1 = \frac{\hat{w} Z_{j_0}}{\hat{v} X_{j_0}} = \hat{w} Z_{j_0}, \quad \hat{e}_{j_0}^2 = \frac{\hat{u} Y_{j_0}}{\hat{w} Z_{j_0} + \hat{\gamma} L_{j_0}}, \quad \hat{e}_{j_0}^o = \frac{\hat{u} Y_{j_0}}{\hat{v} X_{j_0} + \hat{\gamma} L_{j_0}}$$

If $\hat{s}_1 = \hat{s}_2 = 0$, then the optimal solution of (14) is already Pareto optimal, and model (15) does not alter the efficiency scores obtained by (14).

2.2.1. Illustration

We illustrate models (14) and (15) on a two-stage process of type II drawn from Li et al. [19]. The case concerns the assessment of regional R&D process of 30 Provincial Level Regions in China. The stage-1 represents the technology development whereas the stage-2 represents the economic application. The stage-1 inputs are: R&D personnel ($X1$), R&D expenditure ($X2$) and the proportion of regional science and technology funds in regional total financial expenditure ($X3$). The outputs (intermediate measures) of stage-1 which are inputs to stage-2 are: number of patents ($Z1$) and number of papers ($Z2$). The extra input to stage-2 is contract value in technology market (L). The final outputs are GDP ($Y1$), total exports ($Y2$), urban per capita annual income ($Y3$) and gross output of high-tech industry ($Y4$). The reader is referred to [19] for the complete data set.

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Table 4
Results from Li et al. [19] and from model (15) (same as from model (14))

DMU	Li et al. [19]			Model (15)						
	e^1	e^2	$e^o = e^1 \cdot e^2$	E^1	E^2	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^1.\hat{e}^2$	$\hat{e}^o$	DMU
1	1	0.1598	0.1598	1	0.1598	1	0.1598	0.1598	0.1598	1
2	1	0.2489	0.2489	1	0.2489	1	0.2489	0.2489	0.2489	2
3	0.8950	0.5365	0.4802	1	0.5728	0.9314	0.5042	0.4696	0.4696	3
4	0.6774	0.5704	0.3864	0.7426	0.5704	0.7021	0.5300	0.3721	0.3721	4
5	0.6697	0.3895	0.2608	0.6697	0.3895	0.6697	0.3895	0.2608	0.3310	5
6	0.5668	1	0.5668	0.5668	1	0.5668	1	0.5668	0.6137	6
7	1	0.2207	0.2207	1	0.3121	0.9177	0.2298	0.2109	0.2113	7
8	1	1	1	1	1	1	1	1	1	8
9	0.9398	1	0.9398	0.9398	1	0.9398	1	0.9398	0.9534	9
10	1	1	1	1	1	1	1	1	1	10
11	0.8885	0.8351	0.7420	0.8885	0.8351	0.8885	0.8351	0.7420	0.7756	11
12	0.9328	0.2648	0.2470	0.9328	0.2703	0.9278	0.2653	0.2462	0.2566	12
13	0.8493	0.7373	0.6262	0.8504	0.7373	0.8495	0.7364	0.6256	0.6707	13
14	0.9060	0.2816	0.2551	0.9060	0.3360	0.8545	0.2845	0.2431	0.2431	14
15	1	0.3685	0.3685	1	0.3780	0.9921	0.3702	0.3673	0.3689	15
16	0.9225	1	0.9225	0.9225	1	0.9225	1	0.9225	0.9225	16
17	0.5644	0.9914	0.5595	0.5647	1	0.5572	0.9925	0.5531	0.6958	17
18	0.7152	0.4947	0.3538	0.7158	0.5184	0.6986	0.5012	0.3501	0.4158	18
19	0.6671	0.3668	0.2447	0.6969	0.3742	0.6810	0.3583	0.2440	0.2440	19
20	0.4573	1	0.4573	0.4573	1	0.4573	1	0.4573	0.4629	20
21	0.7101	0.8176	0.5806	0.7101	0.8498	0.6854	0.8251	0.5656	0.4573	21
22	0.5708	0.5156	0.2943	0.5864	0.5709	0.5495	0.5340	0.2935	0.3976	22
23	1	0.1941	0.1941	1	0.2509	0.9441	0.1951	0.1842	0.1905	23
24	1	0.4566	0.4566	1	0.4817	0.9758	0.4574	0.4463	0.4517	24
25	1	0.5846	0.5846	1	0.6159	0.9756	0.5915	0.5770	0.5839	25
26	0.7293	0.9171	0.6688	0.9111	0.9541	0.7869	0.8299	0.6530	0.7304	26
27	1	1	1	1	1	1	1	1	1	27
28	0.3599	1	0.3599	0.3599	1	0.3599	1	0.3599	0.3599	28
29	0.4300	1	0.4300	0.4300	1	0.4300	1	0.4300	0.4308	29
30	1	1	1	1	1	1	1	1	1	30

For comparison, we present in Table 4 the results given in [19] and those obtained by model (15). Li et al. [19] calculate the stage-1 and stage-2 efficiency scores parametrically and then they give the overall efficiency as the product of the stage efficiencies, although in their case, the overall efficiency is not readily decomposed to the stage efficiencies, as in the case of the simple structure of Type I [17,20]. However, to be in line with their results, we present the product of the stage efficiencies obtained by our approach as well.

Notice that, for all DMUs, the model (14) provided Pareto optimal solutions. This is validated by the fact that in the second phase program (15), the optimal values of the slacks were $\hat{s}_1 = \hat{s}_2 = 0$. Fourteen out of the 30 units show identical individual efficiency scores. Notice also that $\hat{e}^1.\hat{e}^2 \le e^o$. This is a natural effect of the fact that in [19], among the parametrically generated pairs of stage efficiency scores, the one that shows the maximal squared geometric average is selected. However, as it is explained in Section 2.1, such an approach often assumes implicitly different priorities for the two stages, with one stage arbitrarily favored over the other. Indeed, the stage efficiency scores given in [19] for the units 3, 17, 18, 19, 22 and 26, for example, can be obtained by the weighted variant of model (14) with the couples of weights $(t_1 = 0.256745,\ t_2 = 0.74325)$, $(t_1 = 0.966292,\ t_2 = 0.033708)$$(t_1 = 0.975312,\ t_2 = 0.024688)$, $(t_1 = 0.199731,\ t_2 = 0.800269)$, $(t_1 = 0.779891,\ t_2 = 0.220109)$ and $(t_1 = 0.169132,\ t_2 = 0.830868)$ respectively. The advantage of our approach is that it provides unique and unbiased efficiency scores. However, if it is to assign explicitly different priorities to the two stages, the weighted variant of (14) could be used.

2.3. Type III structure

In the structure of type III, the first stage produces some final outputs K that exit the system, beyond the intermediate measures as depicted in Fig. 1. In this case, the efficiency of the first and the second stage of DMU j are typically defined as follows:

$$e_j^1 = \frac{wZ_j + \mu K_j}{vX_j}, \quad e_j^2 = \frac{uY_j}{wZ_j}$$

The overall efficiency of the system is

$$e_j^o = \frac{uY_j + \mu K_j}{vX_j}$$

On the basis of the above definitions, the bi-objective program for estimating the efficiencies of the two stages is as follows:

$$\begin{aligned}
&\max \frac{wZ_{j_0} + \mu K_{j_0}}{vX_{j_0}} \\
&\max \frac{uY_{j_0}}{wZ_{j_0}} \\
&\text{s.t.} \\
&wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n \\
&uY_j - wZ_j \le 0,\ j = 1, \ldots, n \\
&v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \mu \ge \varepsilon
\end{aligned} \tag{16}$$

Applying the C–C transformation to (16) with respect to the first objective function we get the model below:

$$\begin{aligned}
&\max wZ_{j_0} + \mu K_{j_0} \\
&\max \frac{uY_{j_0}}{wZ_{j_0}} \\
&\text{s.t.} \\
&vX_{j_0} = 1 \\
&wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n \\
&uY_j - wZ_j \le 0,\ j = 1, \ldots, n \\
&v \ge \varepsilon,\ w \ge \varepsilon\ u \ge \varepsilon,\ \mu \ge \varepsilon
\end{aligned} \tag{17}$$

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The following model calculates the stage-1 and stage-2 efficiency scores at a minimum deviation (unweighted L_∞ norm) from their ideal efficiency values:

$$\begin{aligned}&\min \delta\\&\quad \text{s.t.}\\&\quad E^1_{j_0} - wZ_{j_0} - \mu K_{j_0} \le \delta\\&\quad (E^2_{j_0} - \delta)wZ_{j_0} - uY_{j_0} \le 0\\&\quad vX_{j_0} = 1\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \mu \ge \varepsilon,\ \delta \ge 0\end{aligned} \tag{18}$$

The ideal efficiency scores are obtained by solving (16) with one objective function at a time, after transforming it to its linear equivalent. Solving the non-linear model (18) by bisection search for $\delta \in [0, 1]$, we get an optimal solution (δ^*, v^*, w^*, u^*, μ^*), which is weakly Pareto optimal for the MOP (17) and

$$e^{1*}_{j_0} = \frac{w^*Z_{j_0} + \mu^* K_{j_0}}{v^* X_{j_0}} = w^*Z_{j_0} + \mu^* K_{j_0}, \quad e^{2*}_{j_0} = \frac{u^*Y_{j_0}}{w^*Z_{j_0}}$$

The second phase program (19) below provides a Pareto optimal solution to (17):

$$\begin{aligned}&\max s_1 + s_2\\&\quad \text{s.t.}\\&\quad E^1_{j_0} - wZ_{j_0} - \mu K_{j_0} + s_1 = \delta^*\\&\quad (E^2_{j_0} - \delta^*)wZ_{j_0} - uY_{j_0} + w^*Z_{j_0}s_2 = 0\\&\quad vX_{j_0} = 1\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \mu \ge \varepsilon\\&\quad \delta^* \ge s_1 \ge 0,\ \delta^* \ge s_2 \ge 0\end{aligned} \tag{19}$$

Given the optimal solution $(\hat{s}_1, \hat{s}_2, \hat{v}, \hat{w}, \hat{u}, \hat{\mu})$ of (19), the efficiency scores for unit j_0 in the first and the second stage as well as the overall efficiency of the system are respectively:

$$\hat{e}^1_{j_0} = \frac{\hat{w}Z_{j_0} + \hat{\mu}K_{j_0}}{\hat{v}X_{j_0}} = \hat{w}Z_{j_0} + \hat{\mu}K_{j_0}, \quad \hat{e}^2_{j_0} = \frac{\hat{u}Y_{j_0}}{\hat{w}Z_{j_0}},$$

$$\hat{e^o_{j_0}} = \frac{\hat{u}Y_{j_0} + \hat{\mu}K_{j_0}}{\hat{v}X_{j_0}} = \hat{u}Y_{j_0} + \hat{\mu}K_{j_0}$$

If $\hat{s}_1 = \hat{s}_2 = 0$, then the optimal solution of (18) is already Pareto optimal, and model (19) does not alter the efficiency scores obtained by the former.

2.3.1. Illustration

For validation of our computations, we give the data and the results of a synthetic numerical example with 30 DMUs, two inputs ($X1$, $X2$), two intermediate measures ($Z1$, $Z2$), two final outputs from stage-1 ($K1$, $K2$) and two final outputs from stage-2 ($Y1$, $Y2$). The data shown in Table 5 are random and drawn column-wise from a uniform distribution in the intervals given in the last row of Table 5.

For five out of the 30 units (namely, units 11, 14, 17, 19 and 29), the second phase program (19) corrected the efficiency scores derived by the minmax model (18), providing Pareto optimal solutions. For the rest of the units, Pareto optimal solutions were obtained early by model (18).

2.4. Type IV structure

The Type IV structure is the most general two-stage series process. Multiples of this structure in series composes the general multi-stage series process, which will be studied in the next section in the light of our proposed approach. In this case, the efficiency of the first and the second stage of DMU j are defined as follows:

$$e^1_j = \frac{wZ_j + \mu K_j}{vX_j}, \quad e^2_j = \frac{uY_j}{wZ_j + \gamma L_j}$$

The overall system efficiency is

$$e^o_j = \frac{uY_j + \mu K_j}{vX_j + \gamma L_j}$$

The bi-objective program for estimating the efficiencies of the two stages is as follows:

$$\begin{aligned}&\max \frac{wZ_{j_0} + \mu K_{j_0}}{vX_{j_0}}\\&\quad \max \frac{uY_{j_0}}{wZ_{j_0} + \gamma L_{j_0}}\\&\quad \text{s.t.}\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j - \gamma L_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \gamma \ge \varepsilon,\ \mu \ge \varepsilon\end{aligned} \tag{20}$$

Applying the C–C transformation to (20) on the basis of the denominator of the first objective function, we get the following:

$$\begin{aligned}&\max wZ_{j_0} + \mu K_{j_0}\\&\quad \max \frac{uY_{j_0}}{wZ_{j_0} + \gamma L_{j_0}}\\&\quad \text{s.t.}\\&\quad vX_{j_0} = 1\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j - \gamma L_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \gamma \ge \varepsilon,\ \mu \ge \varepsilon\end{aligned} \tag{21}$$

Similarly to the previous structures, the minmax model that calculates the stage-1 and stage-2 efficiency scores at a minimum distance (unweighted L_∞ norm) from their independent counterparts is as follows:

$$\begin{aligned}&\min \delta\\&\quad \text{s.t.}\\&\quad E^1_{j_0} - wZ_{j_0} - \mu K_{j_0} \le \delta\\&\quad (E^2_{j_0} - \delta)(wZ_{j_0} + \gamma L_{j_0}) - uY_{j_0} \le 0\\&\quad vX_{j_0} = 1\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j - \gamma L_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \gamma \ge \varepsilon,\ \mu \ge \varepsilon,\ \delta \ge 0\end{aligned} \tag{22}$$

Solving the model (22) by bisection we get a weakly Pareto optimal solution of the MOP (21) (δ^*, v^*, w^*, u^*, γ^*, μ^*) and

$$e^{1*}_{j_0} = \frac{w^*Z_{j_0} + \mu^* K_{j_0}}{v^* X_{j_0}} = w^*Z_{j_0} + \mu^* K_{j_0}, \quad e^{2*}_{j_0} = \frac{u^*Y_{j_0}}{w^*Z_{j_0} + \gamma^* L_{j_0}}$$

The second phase program (23) below provides a Pareto optimal solution to the MOP (21):

$$\begin{aligned}&\max s_1 + s_2\\&\quad \text{s.t.}\\&\quad E^1_{j_0} - wZ_{j_0} - \mu K_{j_0} + s_1 = \delta^*\\&\quad (E^2_{j_0} - \delta^*)(wZ_{j_0} + \gamma L_{j_0}) - uY_{j_0} + (w^*Z_{j_0} + \gamma^* L_{j_0})s_2 = 0\\&\quad vX_{j_0} = 1\\&\quad wZ_j + \mu K_j - vX_j \le 0,\ j = 1, \ldots, n\\&\quad uY_j - wZ_j - \gamma L_j \le 0,\ j = 1, \ldots, n\\&\quad v \ge \varepsilon,\ w \ge \varepsilon,\ u \ge \varepsilon,\ \gamma \ge \varepsilon,\ \mu \ge \varepsilon\\&\quad \delta^* \ge s_1 \ge 0,\ \delta^* \ge s_2 \ge 0\end{aligned} \tag{23}$$

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Table 5
Synthetic data for type III structure and results obtained from models (18) and (19).

DMU	$X1$	$X2$	$X3$	$Z1$	$Z2$	$K1$	$K2$	$Y1$	$Y2$	E^1	E^2	e^{1*}	e^{2*}	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^o$	DMU
1	71.2	6.3	46.6	61.4	109	54.2	64.3	8.4	36.1	0.9577	0.8675	0.8261	0.7358	0.8261	0.7358	0.6680	1
2	21.3	12.6	59.4	93.5	94.4	31.7	60	5.4	22.5	0.7471	0.4769	0.6897	0.4195	0.6897	0.4195	0.2894	2
3	65.8	16.6	30.1	134.4	57.3	23.1	42.8	12.3	12.8	0.7216	0.8008	0.4995	0.5787	0.4995	0.5787	0.3247	3
4	71.9	12.5	54.2	113	94.8	63.6	35	6.7	36	0.5012	0.7008	0.4988	0.6984	0.4988	0.6984	0.4488	4
5	78.4	5.9	19.5	132	140.8	52.8	19.5	9.1	44	1	0.6409	1	0.6409	1	0.6409	0.6409	5
6	30.4	15	43.3	57.3	100.1	60.4	17.8	7.8	14.1	0.7040	0.4007	0.6297	0.3264	0.6297	0.3264	0.2055	6
7	30.2	11.1	34.5	148	66.6	51.6	43.8	6.9	23.7	0.8818	0.5956	0.7346	0.4484	0.7346	0.4484	0.3294	7
8	11.4	5	33.5	122	69.2	60.6	74.8	2.1	18.5	1	0.4063	1	0.4063	1	0.4063	0.6461	8
9	62.7	15.1	29.2	146.7	102.8	73.6	25.8	7.6	13.6	0.8210	0.2802	0.7551	0.2143	0.7551	0.2143	0.2115	9
10	75.9	11.9	17.4	84.8	141.4	18.9	16.3	13.9	20.1	0.9664	0.4963	0.9657	0.4956	0.9657	0.4956	0.4785	10
11	87.8	19.4	50.9	109.8	53.6	63.2	45.5	11.1	18.3	0.4268	0.7725	0.3723	0.7180	**0.3723**	**0.7420**	0.3723	11
12	17.1	10.8	15.5	98.3	66.4	16.2	31.1	17.8	34.6	1	1	0.9862	0.9862	0.9862	0.9862	0.9785	12
13	48.9	8.9	50.8	133.1	65.6	57.8	64.2	3.4	45.7	0.6282	1	0.5831	0.9549	0.5831	0.9549	0.5714	13
14	82.4	8.1	51	57.1	98.8	73.6	10.5	19.4	38.7	0.7835	1	0.7541	0.9858	**0.7541**	**1**	0.7541	14
15	55.9	14.5	23.4	96.4	88.1	20.8	27.6	4.4	44.6	0.6494	0.9708	0.6382	0.9597	0.6382	0.9597	0.6125	15
16	46.4	10.3	14.8	70.8	127.1	75.7	56.6	12.7	43.5	1	0.9065	1	0.9065	1	0.9065	0.9558	16
17	51.9	7	37.8	119.9	64.1	75.9	23.7	8.8	10.8	0.9267	0.5121	0.9119	0.4973	**0.9119**	**0.5047**	0.9119	17
18	43.2	8.3	51.6	123.2	117	40.6	79.6	9.2	23.2	0.8530	0.4097	0.8192	0.3758	0.8192	0.3758	0.3079	18
19	60.6	14.1	43.8	58	107.5	27.2	62.9	16.2	12.3	0.5695	0.8221	0.5250	0.7776	**0.5250**	**0.7999**	0.5250	19
20	59.7	11.7	22.8	89.9	131.9	15	33	11.7	36.9	0.8400	0.6671	0.8064	0.6335	0.8064	0.6335	0.5108	20
21	26.4	12.2	23.7	142.3	110.7	69.8	47.9	10.2	19.5	1	0.3648	1	0.3648	1	0.3648	0.3648	21
22	70.6	18.3	23.8	93.6	123.1	51.6	43.5	2.3	29	0.7008	0.5350	0.6993	0.5334	0.6993	0.5334	0.3730	22
23	67.8	5.2	46.3	73.7	124.4	50.3	25.2	7.7	12.2	1	0.3134	0.9824	0.2958	0.9824	0.2958	0.6289	23
24	83.5	13.7	47.5	72.9	111.9	34.6	65.5	6.9	43.6	0.5683	0.9473	0.5257	0.9047	0.5257	0.9047	0.5036	24
25	33.8	6	27.6	77.1	74.8	70.5	54.7	9.2	34	1	0.8944	1	0.8944	1	0.8944	0.9735	25
26	81.9	9.4	57.7	131	64.5	14.7	63.5	6.1	21.2	0.5791	0.5489	0.5125	0.4823	0.5125	0.4823	0.3998	26
27	38	12.3	57.6	94.4	98.6	24.1	61.8	14.3	48.6	0.6075	1	0.5587	0.9512	0.5587	0.9512	0.5314	27
28	74.4	7.6	40	141	88.5	73.6	18.9	8.6	13.5	0.8706	0.3625	0.8493	0.3412	0.8493	0.3412	0.7296	28
29	92.2	12.4	45.8	94.5	54.8	23.5	57.9	5.5	11.9	0.5081	0.4043	0.4888	0.3849	**0.4888**	**0.3964**	0.4888	29
30	73.3	14.3	54.8	91.4	53.3	19.2	62.3	4.2	34.7	0.4444	1	0.4444	1	0.4444	1	0.4444	30
	[10,100]	[5,20]	[10,60]	[50,150]	[50,150]	[10,80]	[10,80]	[2,20]	[10,50]								

Once an optimal solution $(\hat{s}_1, \hat{s}_2, \hat{v}, \hat{w}, \hat{u}, \hat{\gamma}, \hat{\mu})$ of (23) is obtained, the efficiency scores for unit j_0 in the first and the second stage as well as the overall efficiency of the system are respectively:

$$\hat{e}^1_{j_0} = \frac{\hat{w}Z_{j_0} + \hat{\mu}K_{j_0}}{\hat{v}X_{j_0}} = \hat{w}Z_{j_0} + \hat{\mu}K_{j_0}, \quad \hat{e}^2_{j_0} = \frac{\hat{\mu}Y_{j_0} + \hat{\gamma}L_{j_0}}{\hat{w}Z_{j_0}}, \quad \hat{e}^o_{j_0} = \frac{\hat{u}Y_{j_0} + \hat{u}K_{j_0}}{\hat{v}X_{j_0} + \hat{\gamma}L_{j_0}}$$

If $\hat{s}_1 = \hat{s}_2 = 0$, then the optimal solution of (22) is already Pareto optimal, and model (23) does not alter the efficiency scores obtained by (22).

2.4.1. Illustration

For testing and validation purposes, we provide the reader with the data (Table 6) and the results (Table 7) of a synthetic numerical example with 30 DMUs, three inputs to stage-1 ($X1$–$X3$), two intermediate measures ($Z1$, $Z2$), to final outputs from stage-1 ($K1$, $K2$), two extra inputs to stage-2 ($L1$, $L2$) and two final outputs from stage-2 ($Y1$, $Y2$). The data exhibited in Table 6 are random and drawn column-wise from a uniform distribution in the intervals given in the last row of Table 6.

As shown in Table 7, in all units except one (namely the unit 24) the second phase program (23) did not alter the efficiency scores obtained by model (22). For unit 24, the second phase program increased the stage-1 efficiency score from 0.9623 to 1 without decreasing the efficiency score of stage-2 (0.8658).

3. Multi-stage processes

A multi-stage series process is actually a multiple of type I–IV structures in series, where links exist only between successive stages. Thus, our developments for two-stage processes can be straightforwardly generalized in multi-stage configurations depicted in Fig. 7.

We adjust the notation as follows:

$j \in J = \{1, \ldots, n\}$: The index set of the n DMUs.
$j_0 \in J$: Denotes the evaluated DMU.
$q = 1, \ldots, Q$ is the index of one of the Q stages.
$X_j^{(q)}$, $q = 1, \ldots, Q$: The vector of stage-q external inputs used by DMU_j.
$Z_j^{(q)}$, $q = 1, \ldots, Q-1$: The vector of intermediate measures passed from stage-q to the next one, for DMU_j.
$Y_j^{(q)}$, $q = 1, \ldots, Q$: The vector of stage-q final outputs produced by DMU_j.
$v^{(q)}$, $q = 1, \ldots, Q$: The vector of weights for the stage-q external inputs.
$w^{(q)}$, $q = 1, \ldots, Q-1$: The vector of weights for the stage-q intermediate measures.
$u^{(q)}$, $q = 1, \ldots, Q$: The vector of weights for the stage-q outputs.
e_j^o: The overall efficiency of DMU_j.
$e_j^{(q)}$, $q = 1, \ldots, Q$: The efficiency of stage-q for DMU_j.
$E_j^{(q)}$, $q = 1, \ldots, Q$: The independent efficiency score of stage-q for DMU_j.

In this general case, the efficiency $e_j^{(q)}$, $q = 1, \ldots, Q$ of each stage is defined as follows:

$$e_j^{(1)} = \frac{u^{(1)}Y_j^{(1)} + w^{(1)}Z_j^{(1)}}{v^{(1)}X_j^{(1)}}$$

$$e_j^{(q)} = \frac{u^{(q)}Y_j^{(q)} + w^{(q)}Z_j^{(q)}}{v^{(q)}X_j^{(q)} + w^{(q-1)}Z_j^{(q-1)}}, \quad q = 2, \ldots, Q-1$$

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Table 6
Synthetic data for type IV structure.

DMU	*X*1	*X*2	*X*3	*Z*1	*Z*2	*K*1	*K*2	*L*1	*L*2	*Y*1	*Y*2
1	22.3	13.2	54.6	110.1	66.1	21.8	44.6	18	31	13.3	12.5
2	68.3	8.3	15.8	75.4	116.4	19.8	12	19.6	25.8	2.4	18.2
3	52	19.2	31.2	94.3	59.9	47.3	47.4	11.5	22.5	2.3	36
4	31.8	12	40.3	66.4	127.2	10.5	35.8	16.8	37.1	3	19.5
5	95.3	12	29	108.9	52.3	15	22.5	14	27.2	15.8	16.7
6	52.8	6.1	22.6	102.4	78.8	69.6	27	14.8	44.9	12.6	20.4
7	50.5	9.3	48.7	124.6	120.6	52.2	49.8	5.9	38.5	9.5	20.5
8	80.1	17.4	58.4	64.5	131.2	37.7	14.6	10.3	65.6	16.7	39.9
9	53.9	14	36.9	129.8	122.1	60.9	24.1	11.9	49.5	16.8	15
10	20.9	9.5	48.8	66.4	132.5	12.2	68.7	10.1	54.5	10	28.3
11	82.5	7.1	16.8	71.9	138.9	47.7	60.7	5.6	19.1	19.7	33.6
12	27	10.6	25.6	51.9	84.4	47.3	63.3	11	39.6	12.2	43.7
13	49.6	10.7	20.6	125.5	97.3	15.3	32.6	17.7	38.9	18.9	44.7
14	55.7	19.4	46.6	91.5	117.3	79	60.3	11.8	26.4	7.5	38.7
15	55.1	18.2	52.5	90.1	61	12.2	24.9	17	33.5	17.2	43.9
16	66.3	8	34.9	131.1	63.7	57	30.7	10.7	52.5	11.2	15.5
17	93.3	6.3	43.5	53.5	133.9	38.6	32.1	13.4	45	19.7	15.4
18	10.8	11.9	31.5	118.7	89.4	34.9	23.6	11	67.3	8	20.5
19	98.5	6.8	21.3	75.4	133	28	28.9	16.1	26.7	9.5	20.3
20	27.8	17.1	24.9	81	52.2	30.6	14.3	16.9	35.3	17.4	15.4
21	42	7.2	59.7	98.4	147.5	29.2	39.4	14.8	42.3	10.7	44.5
22	98.7	8.5	51	132.8	60.6	27.3	69.3	19.8	61.9	19.9	33.3
23	53.5	15.6	25.7	93.5	121.6	31.3	34.6	19.7	56.5	13	47.6
24	25.1	16.7	56.8	81.6	145.6	62.1	74.8	11.7	17.4	7.6	29.9
25	96.3	15.3	45.1	120.5	133.6	25.7	56.8	19.7	16.9	14.9	38.5
26	97.9	6.8	53.1	103.8	89.8	45.7	49.6	17.7	56.3	4.9	12.5
27	37.4	15	15.5	63.1	128.2	53.1	22	5.5	57.7	5.8	11.8
28	70	12.8	21.5	126.1	97.2	28.3	44.3	11.4	56.7	4.9	47.2
29	24	5.8	33.8	91.2	82.6	73.7	76.2	19.4	42.4	7.8	10.3
30	48.6	18.6	55.9	126	73.8	15.4	57.6	17.1	76.2	13.2	25.6
	[10,100]	[5,20]	[10,60]	[50,150]	[50,150]	[10,80]	[10,80]	[5,20]	[10,80]	[2,20]	[10,50]

Table 7
Results obtained from models (22) and (23) applied to the data of Table 6.

DMU	E^1	E^2	e^{1*}	e^{2*}	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^o$
1	0.7338	0.7327	0.6751	0.6739	0.6751	0.6739	0.5573
2	1	0.4751	0.9868	0.4619	0.9868	0.4619	0.4643
3	0.7608	1	0.7287	0.9679	0.7287	0.9679	0.7805
4	0.8938	0.4268	0.8280	0.3610	0.8280	0.3610	0.3361
5	0.6914	1	0.5488	0.8574	0.5488	0.8574	0.5360
6	1	0.6708	1	0.6708	1	0.6708	0.8084
7	0.8812	0.6316	0.8529	0.6033	0.8529	0.6033	0.5606
8	0.5496	0.9796	0.4727	0.9028	0.4727	0.9028	0.4842
9	0.8656	0.7555	0.8298	0.7198	0.8298	0.7198	0.6305
10	1	0.6316	1	0.6316	1	0.6316	0.8970
11	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1
13	1	0.8690	0.9715	0.8405	0.9715	0.8405	0.8321
14	0.6687	0.9337	0.6686	0.9336	0.6686	0.9336	0.6687
15	0.4215	1	0.4119	0.9904	0.4119	0.9904	0.6253
16	0.9813	0.7660	0.7815	0.5662	0.7815	0.5662	0.4829
17	1	1	0.9344	0.9344	0.9344	0.9344	0.8910
18	1	0.5189	1	0.5189	1	0.5189	0.5189
19	1	0.5096	0.9858	0.4954	0.9858	0.4954	0.4922
20	0.7676	1	0.7067	0.9390	0.7067	0.9390	0.7086
21	1	0.7871	0.9975	0.7846	0.9975	0.7846	0.7838
22	0.9591	1	0.7993	0.8403	0.7993	0.8403	0.6978
23	0.8255	0.7260	0.8178	0.7184	0.8178	0.7184	0.6697
24	1	0.9035	0.9623	0.8658	**1**	**0.8658**	0.8658
25	0.6613	1	0.6613	1	0.6613	1	0.7661
26	0.9447	0.2427	0.9343	0.2323	0.9343	0.2323	0.2395
27	1	0.3791	1	0.3791	1	0.3791	0.5963
28	1	1	0.9030	0.9030	0.9030	0.9030	0.8742
29	1	0.3762	1	0.3762	1	0.3762	0.3762
30	0.6246	0.6813	0.5512	0.6079	0.5512	0.6079	0.4320

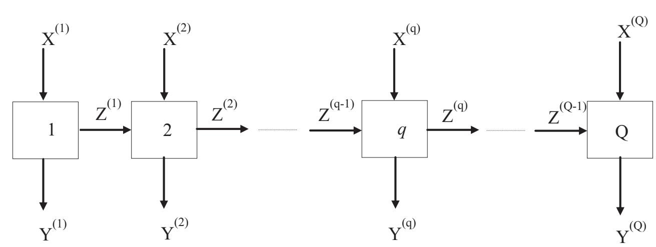

Fig. 7. General multi-stage series process.

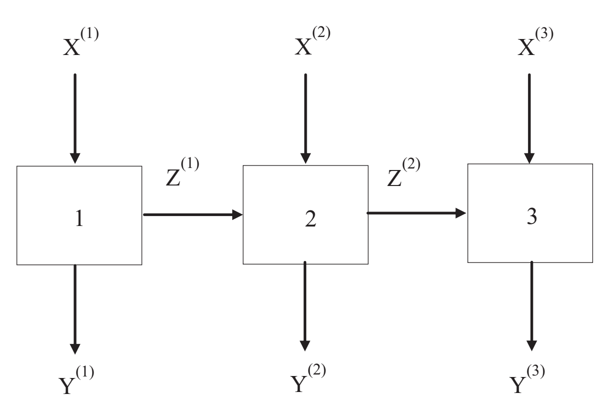

Fig. 8. A three-stage process.

Table 8
Synthetic data for the multi-stage process of Fig. 8.

DMU	$X_1^{(1)}$	$X_2^{(1)}$	$Y_1^{(1)}$	$Y_2^{(1)}$	$Z_1^{(1)}$	$Z_2^{(1)}$	$X_1^{(2)}$	$X_2^{(2)}$	$Y_1^{(2)}$	$Y_2^{(2)}$	$Z_1^{(2)}$	$Z_2^{(2)}$	$X_1^{(3)}$	$X_2^{(3)}$	$Y_1^{(3)}$	$Y_2^{(3)}$
1	56.7	8.5	10.9	37.5	39	47.3	54.7	47	36.8	23.1	21.4	25.7	58.2	49.7	36.4	37.2
2	50.9	7.7	3.3	42.8	73	36.7	16.9	57.7	34	11	36.9	35.1	12.1	37.2	43.4	58.5
3	16.6	17.4	18	32.1	72.4	18.3	29.5	11.6	12.2	37.8	36.6	35.9	58.6	44.3	26.6	50.5
4	67.6	16.5	3.2	48.5	63.5	36.2	56.4	27.8	12.3	25.7	14.9	26.8	19.5	54.7	47.2	20.3
5	89.7	19	9.9	44.3	20.7	12.9	55.9	43.1	16.1	20.4	26.3	26.9	43.4	12.7	58.9	47.2
6	38.4	6.6	16.9	29.4	11.7	26.3	45.7	24.1	10.8	38	21	33.3	39.3	25.2	33.4	48.2
7	87.7	7.7	9.1	33.2	54.8	36.7	40.9	21.5	27.4	28	38.7	41.2	43.8	12.3	63.2	45.8
8	37.7	6.5	13	30.5	34.1	47.3	27.2	45.6	43.3	20.8	21.3	57.3	28.1	19.8	24.1	42.1
9	80.9	12.3	16.7	46	85.2	42.4	56.8	41.2	34.7	34.4	45.8	44.7	41	46	41.7	28.7
10	46.4	7.9	18	44.4	88.5	29.4	16.2	39.5	30.8	23.6	43.1	27	50.6	46.1	20.2	50.9
11	43.1	18.4	18.8	32.2	32.9	40.3	46.5	43	44.6	24.4	25.6	50.8	11	53.9	32.1	29.1
12	27.4	6.5	5.4	42.8	74.1	26.7	42.3	12.4	13.9	28.8	29.9	12.4	14.2	39.1	20.1	34.8
13	20.1	5.7	6.7	52.8	81.7	48.9	51.7	27.4	46.3	17.9	37.8	8.3	58.7	13.5	30.4	55.6
14	34.7	13.4	18.2	46.4	57.8	49.5	29.9	32.6	14.3	13.4	43.4	9.6	42.6	56.1	27.8	54.3
15	68.1	16.6	12.7	43.2	80.7	44.6	47.5	22	30.7	25.3	34.4	14	21.6	50	34.7	36.1
16	72.6	9.7	11.1	19.4	85.5	25.6	51.8	45.8	15.7	35.3	33	22.8	30.2	24.3	29.6	32.7
17	65.5	7.7	13	30.6	53.9	28.2	26.1	52.8	32.4	19.2	23	21.6	16.1	37.2	27.6	44.3
18	10.8	10.1	16.7	38.6	68.3	19.9	37.6	24.1	10.2	8.9	28.3	5.6	23.4	59.2	52.9	56.4
19	77.5	8.2	11.6	33.4	56.1	41.4	59	46.6	40.7	20.5	38.6	34.7	22.9	45.8	69.6	56.4
20	83.8	12.7	5.6	17	12.1	45.3	37.5	16.9	43.9	15.5	45.4	10.2	26.6	51.9	71.7	43.7
21	71.7	18.6	10.2	24.1	45.7	46.5	26.5	51.8	46.7	19	38.8	13.1	17.6	31.7	32.2	33.3
22	13.4	14.4	9.7	48.4	61.7	32.3	41	16.9	49.5	34.2	10.7	39.7	27.4	33.5	46.5	54.1
23	40.3	6.5	19.4	15.6	51.7	34	28	39.4	30.2	19.1	37	52.3	16.1	38	40.7	37.7
24	66.3	10.9	13.2	49.5	39.8	16	47.8	28.3	20.9	18.7	27.5	58.6	54.2	23.5	48.8	56.2
25	68.4	5.8	14.5	18.1	85	46	30.7	50.3	14	17.6	27.5	36.4	14.7	47.5	34.6	21.3
26	27.9	12.5	15	41.8	76.4	28	34.6	35.2	30.3	9.9	14.7	59.8	56.5	35.2	23.8	41.3
27	31.5	11.5	8.2	35	77.9	18.2	44.7	34.5	33.4	14.1	42.6	35.4	30	42.3	44	48.7
28	63.8	20	11.3	23.7	39.8	46	58.6	53.9	40.5	8	23	33.4	12.4	25.4	29.6	27.2
29	48.2	17.2	12	37.9	57.5	40.5	26.4	27.7	13.3	20	19.8	23.2	27.1	16.9	21.4	33.5
30	59.9	12.3	4.8	19.9	79.8	45.3	51.9	32.5	36.5	14	23.7	28.7	46.8	33.8	72.5	27.5
	[10,90]	[5,20]	[2,20]	[15,55]	[10,90]	[10,50]	[10,60]	[10,60]	[10,50]	[5,40]	[10,50]	[5,60]	[10,60]	[10,60]	[20,75]	[20,60]

Table 9
Results obtained from models (25) and (26) applied to the data of Table 8.

DMU	E^1	E^2	E^3	e^1*	e^2*	e^3*	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^3$	$\hat{e}^o$	DMU
1	0.6979	0.7085	0.6474	0.6692	0.6798	0.6188	0.6692	0.6798	0.6188	0.6356	1
2	0.6594	1	1	0.6594	1	1	0.6594	1	1	0.8133	2
3	0.7012	1	0.6006	0.7012	1	0.6006	0.7012	1	0.6006	0.6891	3
4	0.3173	0.4927	1	0.2963	0.4717	0.9790	**0.3068**	**0.4717**	**0.9985**	0.3003	4
5	0.3083	1	1	0.3083	1	1	0.3083	1	1	0.3083	5
6	1	1	0.8725	1	1	0.8724	1	1	0.8724	0.9933	6
7	0.6428	0.9838	1	0.6275	0.9685	0.9847	**0.6275**	**0.9685**	**0.9983**	0.8213	7
8	0.9935	1	0.9554	0.9527	0.9593	0.9147	**0.9935**	**0.9593**	**0.9350**	0.9593	8
9	0.6355	0.8040	0.4696	0.6340	0.8025	0.4681	**0.6347**	**0.8025**	**0.4681**	0.6054	9
10	1	1	0.6376	1	1	0.6376	1	1	0.6376	0.7888	10
11	0.5796	1	0.8851	0.5322	0.9526	0.8378	0.5322	0.9526	0.8378	0.5421	11
12	0.7939	0.8150	0.8203	0.7534	0.7746	0.7798	**0.7939**	**0.7746**	**0.8249**	0.7746	12
13	1	0.8043	1	1	0.8043	1	1	0.8043	1	0.9403	13
14	0.8064	0.8718	0.7640	0.6911	0.7566	0.6488	**0.7178**	**0.7566**	**0.6488**	0.6566	14
15	0.4241	0.6843	0.6668	0.3929	0.6531	0.6356	0.3929	0.6531	0.6356	0.3929	15
16	0.6169	0.7314	0.6621	0.5491	0.6635	0.5942	**0.5491**	**0.6635**	**0.6082**	0.5454	16
17	0.7395	0.9225	0.9456	0.7380	0.9210	0.9440	**0.7380**	**0.9210**	**0.9441**	0.7391	17
18	1	0.7168	1	0.9779	0.6868	0.9700	**1**	**0.6868**	**0.9700**	0.8158	18
19	0.6961	0.8030	1	0.6887	0.7956	0.9926	**0.6887**	**0.7956**	**0.9999**	0.6784	19
20	0.4158	1	1	0.4158	1	1	0.4158	1	1	0.6516	20
21	0.3437	1	0.8453	0.3437	1	0.8453	0.3437	1	0.8453	0.9219	21
22	1	1	1	1	1	1	1	1	1	1	22
23	1	1	0.7844	1	1	0.7844	1	1	0.7844	0.9360	23
24	0.6497	1	0.9073	0.6449	0.9951	0.9024	0.6449	0.9951	0.9024	0.6454	24
25	1	0.6358	0.7559	0.9700	0.6058	0.7259	**1**	**0.6058**	**0.7259**	0.7349	25
26	0.7068	1	0.6771	0.5676	0.8608	0.5379	**0.6752**	**0.8608**	**0.5379**	0.5908	26
27	0.5689	1	0.6982	0.4697	0.9008	0.5990	**0.4697**	**0.9008**	**0.6615**	0.4713	27
28	0.3655	0.7667	0.7944	0.3359	0.7372	0.7649	**0.3359**	**0.7372**	**0.7824**	0.3095	28
29	0.4378	0.6626	0.8439	0.4050	0.6297	0.8110	**0.4050**	**0.6298**	**0.8132**	0.4048	29
30	0.4510	0.6185	1	0.4373	0.6048	0.9863	**0.4373**	**0.6048**	**0.9999**	0.3540	30

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$$e_j^{(Q)} = \frac{u^{(Q)} Y_j^{(Q)}}{v^{(Q)} X_j^{(Q)} + w^{(Q-1)} Z_j^{(Q-1)}}$$

Model (24) below is a multi-objective program with Q objective functions, each representing the efficiency of stage-q, $q=1,\ldots,Q$.

$$\begin{aligned}
&\max \frac{u^{(1)} Y_{j_0}^{(1)} + w^{(1)} Z_{j_0}^{(1)}}{v^{(1)} X_{j_0}^{(1)}} \\
&\max \frac{u^{(q)} Y_{j_0}^{(q)} + w^{(q)} Z_{j_0}^{(q)}}{v^{(q)} X_{j_0}^{(q)} + w^{(q-1)} Z_{j_0}^{(q-1)}}, \quad q=2,\ldots,Q-1 \\
&\max \frac{u^{(Q)} Y_{j_0}^{(Q)}}{v^{(Q)} X_{j_0}^{(Q)} + w^{(Q-1)} Z_{j_0}^{(Q-1)}} \\
&\text{s.t.} \\
&u^{(1)} Y_j^{(1)} + w^{(1)} Z_j^{(1)} - v^{(1)} X_j^{(1)} \le 0, \; j=1,\ldots,n \\
&u^{(q)} Y_j^{(q)} + w^{(q)} Z_j^{(q)} - v^{(q)} X_j^{(q)} - w^{(q-1)} Z_j^{(q-1)} \le 0, \; j=1,\ldots,n, \\
&q=2,\ldots,Q-1 \\
&u^{(Q)} Y_j^{(Q)} - v^{(Q)} X_j^{(Q)} - w^{(Q-1)} Z_j^{(Q-1)} \le 0, \; j=1,\ldots,n \\
&v^{(q)} \ge \varepsilon, u^{(q)} \ge \varepsilon, \; q=1,\ldots,Q \\
&w^{(q)} \ge \varepsilon, \; q=1,\ldots,Q-1
\end{aligned} \tag{24}$$

The ideal values (independent efficiency scores) $E_{j_0}^{(q)}$, $q=1,\ldots,Q$ of the Q stages are obtained by considering each objective function separately and solving the linear equivalent of model (24) derived by the C–C transformation. Given the ideal efficiency scores that each stage attains when considered independently from the others, the program (25) below provides a weakly Pareto optimal solution to the MOP (24) and estimates efficiency scores for the Q stages as close as possible to their ideal counterparts with respect to the unweighted L_∞ norm. Model (25) below is derived by applying the C–C transformation to (24) on the basis of the denominator of the first objective function and is solved by bisection search.

$$\begin{aligned}
&\min \delta \\
&\text{s.t.} \\
&E_{j_0}^{(1)} - u^{(1)} Y_{j_0}^{(1)} - w^{(1)} Z_{j_0}^{(1)} \le \delta \\
&(E_{j_0}^{(q)} - \delta)(v^{(q)} X_{j_0}^{(q)} + w^{(q-1)} Z_{j_0}^{(q-1)}) - u^{(q)} Y_{j_0}^{(q)} - w^{(q)} Z_{j_0}^{(q)} \le 0, \\
&q=2,\ldots,Q-1 \\
&(E_{j_0}^{(Q)} - \delta)(v^{(Q)} X_{j_0}^{(Q)} + w^{(Q-1)} Z_{j_0}^{(Q-1)}) - u^{(Q)} Y_{j_0}^{(Q)} \le 0 \\
&v^{(1)} X_{j_0}^{(1)} = 1 \\
&u^{(1)} Y_j^{(1)} + w^{(1)} Z_j^{(1)} - v^{(1)} X_j^{(1)} \le 0, \; j=1,\ldots,n \\
&u^{(q)} Y_j^{(q)} + w^{(q)} Z_j^{(q)} - v^{(q)} X_j^{(q)} - w^{(q-1)} Z_j^{(q-1)} \le 0, \; j=1,\ldots,n, \\
&q=2,\ldots,Q-1 \\
&u^{(Q)} Y_j^{(Q)} - v^{(Q)} X_j^{(Q)} - w^{(Q-1)} Z_j^{(Q-1)} \le 0, \; j=1,\ldots,n \\
&v^{(q)} \ge \varepsilon, u^{(q)} \ge \varepsilon, \; q=1,\ldots,Q \\
&w^{(q)} \ge \varepsilon, \; q=1,\ldots,Q-1 \\
&\delta \ge 0
\end{aligned} \tag{25}$$

Let (δ^*, $v^{*(q)}$, $q=1,\ldots,Q$, $w^{*(q)}$, $q=1,\ldots,Q-1$, $u^{*(q)}$, $q=1,\ldots,Q$) be an optimal solution of model (25), which is weakly Pareto optimal for (24) and

$$e_{j_0}^{*(1)} = \frac{u^{*(1)} Y_{j_0}^{(1)} + w^{*(1)} Z_{j_0}^{(1)}}{v^{*(1)} X_{j_0}^{(1)}} = u^{*(1)} Y_{j_0}^{(1)} + w^{*(1)} Z_{j_0}^{(1)}$$

$$e_{j_0}^{*(q)} = \frac{u^{*(q)} Y_{j_0}^{(q)} + w^{*(q)} Z_{j_0}^{(q)}}{v^{*(q)} X_{j_0}^{(q)} + w^{*(q-1)} Z_{j_0}^{(q-1)}}, \quad q=2,\ldots,Q-1$$

$$e_{j_0}^{*(Q)} = \frac{u^{*(Q)} Y_{j_0}^{(Q)}}{v^{*(Q)} X_{j_0}^{(Q)} + w^{*(Q-1)} Z_{j_0}^{(Q-1)}}$$

The second phase program that provides a Pareto optimal solution for the MOP (24) is as follows:

$$\begin{aligned}
&\max \sum_{q=1}^{Q} s^{(q)} \\
&\text{s.t.} \\
&E_{j_0}^{(1)} - u^{(1)} Y_{j_0}^{(1)} - w^{(1)} Z_{j_0}^{(1)} + s^{(1)} = \delta^* \\
&(E_{j_0}^{(q)} - \delta^*)(v^{(q)} X_{j_0}^{(q)} + w^{(q-1)} Z_{j_0}^{(q-1)}) - u^{(q)} Y_{j_0}^{(q)} - w^{(q)} Z_{j_0}^{(q)} \\
&+ (v^{*(q)} X_{j_0}^{(q)} + w^{*(q-1)} Z_{j_0}^{(q-1)}) s^{(q)} = 0, \; q=2,\ldots,Q-1 \\
&(E_{j_0}^{(Q)} - \delta^*)(v^{(Q)} X_{j_0}^{(Q)} + w^{(Q-1)} Z_{j_0}^{(Q-1)}) - u^{(Q)} Y_{j_0}^{(Q)} \\
&+ (v^{*(Q)} X_{j_0}^{(Q)} + w^{*(Q-1)} Z_{j_0}^{(Q-1)}) s^{(Q)} = 0 \\
&v^{(1)} X_{j_0}^{(1)} = 1 \\
&u^{(1)} Y_j^{(1)} + w^{(1)} Z_j^{(1)} - v^{(1)} X_j^{(1)} \le 0, \; j=1,\ldots,n \\
&u^{(q)} Y_j^{(q)} + w^{(q)} Z_j^{(q)} - v^{(q)} X_j^{(q)} - w^{(q-1)} Z_j^{(q-1)} \le 0, \\
&j=1,\ldots,n, \; q=2,\ldots,Q-1 \\
&u^{(Q)} Y_j^{(Q)} - v^{(Q)} X_j^{(Q)} - w^{(Q-1)} Z_j^{(Q-1)} \le 0, \; j=1,\ldots,n \\
&v^{(q)} \ge \varepsilon, u^{(q)} \ge \varepsilon, \; q=1,\ldots,Q \\
&w^{(q)} \ge \varepsilon, \; q=1,\ldots,Q-1 \\
&\delta^* \ge s^{(q)} \ge 0, \; q=1,\ldots,Q
\end{aligned} \tag{26}$$

Given an optimal solution ($\hat{v}^{(q)}$, $\hat{u}^{(q)}, \hat{s}^{(q)}; q=1,\ldots,Q$, $\hat{w}^{(q)}$, $q=1,\ldots,Q-1$) of (26) the stage efficiency scores for the evaluated unit j_0 and the overall system efficiency are:

$$\hat{e}_{j_0}^{(1)} = \frac{}{\hat{u}^{(1)}} Y_{j_0}^{(1)} + \hat{w}^{(1)} Z_{j_0}^{(1)} \hat{v}^{(1)} X_{j_0}^{(1)} = \hat{u}^{(1)} Y_{j_0}^{(1)} + \hat{w}^{(1)} Z_{j_0}^{(1)}$$

$$\hat{e}_{j_0}^{(q)} = \frac{\hat{u}^{(q)} Y_{j_0}^{(q)} + \hat{w}^{(q)} Z_{j_0}^{(q)}}{\hat{v}^{(q)} X_{j_0}^{(q)} + \hat{w}^{(q-1)} Z_{j_0}^{(q-1)}}, \quad q=2,\ldots,Q-1$$

$$\hat{e}_{j_0}^{(Q)} = \frac{\hat{u}^{(Q)} Y_{j_0}^{(Q)}}{\hat{v}^{(Q)} X_{j_0}^{(Q)} + \hat{w}^{(Q-1)} Z_{j_0}^{(Q-1)}}$$

$$\hat{e}_{j_0}^{o} = \frac{\sum_{q=1}^{Q} Y_{j_0}^{(q)}}{\sum_{q=1}^{Q} X_{j_0}^{(q)}}$$

3.1. Illustration

For testing and validation to be made possible, we provide a synthetic example with 30 DMUs operating as three-stage processes, as depicted in Fig. 8. The randomly generated data and the results obtained by solving model (26) for each DMU are given in Tables 8 and 9 respectively.

The bold figures in columns 8–10 of Table 9 indicate the units, whose final Pareto optimal efficiency scores were obtained in the second phase.

4. Conclusion

We presented in this paper a novel network DEA approach to efficiency assessment in series multi-stage processes. Actually, it is a multi-objective programming approach that employs the L_∞ norm as distance measure to locate the stage efficiency scores as close as possible to their ideal values. Our approach is general, in the sense that it can handle series multi-stage processes of any type. It is exact, as it provides unique efficiency scores and it is neutral as it treats the different stages equivalently. Also, it responds accurately to any different weighting scheme for the

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stages, by driving the efficiency assessments accordingly. The extendibility of the models to variable returns-to-scale (VRS) situations as well as the applicability of the proposed approach to more complex non-series structures are subjects for further research.

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