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Ali Emrouznejad
Emilyn Cabanda *Editors*

Managing Service Productivity

Using Frontier Efficiency Methodologies and Multicriteria Decision Making for Improving Service Performance

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To our fathers

Preface

Frontier efficiency methodologies are classified into two types: *deterministic production frontier and stochastic production frontier*. *Frontier* refers to the maximum limit which represents the best-practice approaches to production. *Efficiency* is the use of maximum outputs produced from a given mix of inputs. *Stochastic production frontier* [i.e., Stochastic Frontier Analysis (*SFA*)] allows technical inefficiency effects, can account statistical noise in the measurement of efficiency, and also specifies a functional form for the production (e.g., cost function). *Deterministic production frontier*, such as *data envelopment analysis* (DEA), is a goal programming approach, which assumes that any deviations of decision-making units (DMUs) from the frontier are due to technical inefficiency. A key advantage of this approach over SFA is that it more easily accommodates both multiple inputs and multiple outputs and since it is a nonparametric approach prior aggregation of the inputs or outputs is not necessary. Further, a specific functional form for the production process does not need to be imposed on the model (as is required in the use of the SFA approach).

Since its introduction in 1978, DEA has become one of the preeminent nonparametric methods for measuring efficiency and productivity of DMUs. DEA is a linear programming technique which determines the best-practice frontier from a set of peers (DMUs) and measures efficiency between best-practice and observed units using multiple inputs and outputs. DEA models are now employed routinely in areas that range from assessment of public sectors such as hospitals and health care systems and schools and universities to private sectors such as banks and financial institutions. The advantage of DEA is to accommodate multiple inputs and multiple outputs for measuring the relative efficiencies of a set of homogeneous decision-making units without a priori assumption of profit maximization and cost minimization. DEA models are useful for performance evaluation and improvement of DMUs, including the multidimensional aspects of service efficiency issues and operations that can help managers improve service performance.

On the other hand, multi-criteria decision-making (MCDM) models require complex optimization problems using multiple objectives than using a single objective of either maximizing profit or minimizing cost. The multiple criteria

DEA and other goal programming models are examples wherein the decision maker can use multiple outputs and multiple inputs to examine service performance and improvement. This book also reveals how DEA is used in multi-criteria decision making and as a benchmarking tool.

Both DEA and MCDM have been frequently applied for measuring efficiency and productivity of service industries. Service sectors include financial services (banking, insurance, securities, fund management), professional services (accounting, legal, engineering, architecture), health services, education services, environmental services, energy services, logistics, tourism, information technology, telecommunications, transport, distribution, standards and conformance, audio-visual, media, entertainment, cultural, and other business services.

With the exception of some basic notions in DEA, this book is completely self-contained. Important concepts and applications in measuring efficiency of the service sector are carefully motivated and introduced. Specifically, we have excluded any technical material that does not contribute directly to the understanding of measuring efficiency with DEA. Many other excellent textbooks are available today that discuss DEA in much more technical detail than is provided here. This book is aimed at upper-level undergraduate as well as beginning graduate students who want to learn more about measuring and managing service productivity with DEA and other MCDM techniques, or who are pursuing research in DEA and its applications.

The main objective of this book is to provide the necessary background to work with existing DEA models. Once the material in this book has been mastered, the reader will be able to apply DEA models to his or her problems for measuring comparative efficiency of decision-making units in any service industry.

To facilitate this goal, the first chapter provides a literature review and summary of the current research in DEA with a focus on the service sector. In this introductory chapter we present a classification scheme with seven main primary categories in service industry, namely, education, hospital and healthcare, tourism, banking and finance, information technology and media services, transportations, and utilities. We discuss each classification scheme and group selected DEA papers published in the literature over the past three decades. Finally, we provide information on the use of Performance Improvement Management Software (PIM-DEA). A free limited version of this software and downloading procedure is also included in this book. This advanced DEA software enables you to make the best possible analysis of your data, using the latest theoretical developments in DEA. PIM-DEA software gives you the capacity to assess efficiency and productivity, set targets, identify benchmarks, and much more allowing you to truly manage the performance of any service industry. PIM-DEA is easy to use and powerful, and it has an extensive range of the most up-to-date DEA models, which can handle large sets of data.

This is followed by chapter "Development of Assessment Model for Research Efficiency of Universities," where **Jong-Woun Youn and Kwangtae Park** argue that the research in university is an essential part for national competitiveness and the foundation of knowledge and information of a society. This chapter assumes that the effective operation of limited resources by size of universities would be the plan for maximizing their effectiveness and suggests a grouping of similar universities by establishing a new classifying system. Based on the classifying system proposed in this chapter, four models including High Efficiency Expanding Model (HEEM), High Efficiency Stable Model (HESM), Low Efficiency Stable Model (LESM), and Low Efficiency Expanding Model (LEEM) are suggested through a practical analysis.

In the same content of education, **Dimitris Sotiros, Yannis G. Smirlis, and Dimitris K. Despotis** present an alternative method to assess the quality and extent of research in higher education in chapter "Incorporating Intra- and Inter-Input/Output Weight Restrictions in Piecewise Linear DEA: An Application to the Assessment of the Research Activity in Higher Education." They proposed an extension of Piecewise Linear DEA to value-based piecewise linear DEA that incorporates value judgments and allows common treatment for intra- and inter-input/output weight restrictions. Value-based piecewise Linear DEA further enables a better expression of individual preferences, enhances the model with the fully units invariance property, and also resolves the discontinuity issue that exists in the original Piecewise Linear DEA model.

The next two chapters are examples of use of DEA in health care efficiency. **Felix Masiye, Chrispin Mphukaa, and Ali Emrouznejad**, in chapter "Estimating the Efficiency of Healthcare Facilities Providing HIV/Aids Treatment in Zambia: A Data Envelopment Approach," discuss that many countries in Sub-Saharan Africa face a key challenge of sustaining high levels of coverage of AIDS treatment under prospects of dwindling global resources for *HIV/AIDS* treatment. Policy debate in *HIV/AIDS* is increasingly paying more focus on efficiency in the use of available resources. The aim of this chapter is to provide a framework to estimate short-term technical efficiency of HIV/AIDS treatment facilities using DEA. An application in Zambia shows the applicability of the proposed model.

In the same area of health efficiency, a benchmarking approach based on closest targets is given in chapter "Benchmarking in Healthcare: An Approach Based on Closest Targets" where **Juan Aparicio, Fernando Borras, Lidia Ortiz, and Jesus T. Pastor** examine the process of benchmarking in hospital performance. In particular, this chapter shows that the determination of closest targets as a benchmarking technique has significant advantages over traditional DEA methods for signaling keys for the inefficient hospitals to improve their performance. In doing so, this chapter uses a sample of hospitals, located in the eastern region of Spain. Further, some guidance in relation to determining potential improvement targets for each of the inefficient hospitals is given.

Services are becoming increasingly important to the developed and developing economies. However, evidence shows that as production moves from agriculture and manufacturing to service- and knowledge-based economies, productivity growth rates have declined. To date, there are no clear indicators for quantifying productivity for service and network based firms. This raises the question: *How can productivity be measured for service and network based firms*? **Moira Scerri and Renu Agarwal**, in chapter "Service Enterprise Productivity in Action (SEPIA)," present a systems view of productivity, which is organized into five sections: overview of productivity; current measures of productivity using KLEMS; existing service productivity models; service enterprise productivity in action (SEPIA) model; and new measures for service enterprise productivity. The key contribution of this chapter involves the operationalization of the SEPIA model and an illustration of the model through the use of an industry example.

This is followed by measuring good governance in chapter "Using Data Envelopment Analysis to Measure Good Governance" where **Rouselle Lavado, Emilyn Cabanda, Jessamyn Encarnacion, Severa de Costo**, **and Jose Ramon Albert** provide an estimate of good governance index using the DEA with evidence from Philippine provinces. This chapter illustrates how DEA can be used to provide insights on how provinces can improve on various indicators of governance. Aside from identifying peers, DEA is also able to estimate targets, which can serve as a guide for central governments in holding provinces accountable. This chapter shows that DEA is not used only for efficiency measurement but also applied to other applications in benchmarking and index generation, including nonprofit sectors such as public agencies.

A DEA-based methodology is developed in chapter "Measuring the Performance of Service Organizations and the Effects of Downsizing on Performance: Evidence from the Greek Citizen Service Centers" to measure the performance of not-for-profit and for-profit service organizations. **Panagiotis D. Zervopoulos** proposes a methodology that can incorporate endogenous and exogenous variables in the production process, which are directly or inversely related. This methodology always identifies reference units that are qualified in all of the dimensions of performance. In addition, it defines appropriate changes to the resources that are used by the low-performing units to enable them to become qualified in all facets of performance at the optimal condition. The methodology that is developed in this chapter is applied to public organizations, which are in charge of the provision of administrative services to citizens, in two instances: before and after the implementation of downsizing as part of the public management reform agenda. The results obtained from the assessment methodology are the basis for the analysis of the impact of structural reform, and particularly of downsizing, on the performance of public service organizations.

Luciana Yeung, in chapter "Measuring Efficiency of Courts: An Assessment of Brazilian Courts Productivity," develops a DEA framework for measuring efficiency in the Judiciary, specifically in State Courts with an illustration from Brazil. The chapter argues that both inefficient and unstable units could use DEA results to improve their management and to achieve better results in their efficiency, productivity, and effectiveness in the delivery of judicial services.

This is followed by an application of cost-efficiency and market power in chapter "Cost Efficiency and Market Power: A Test of Quiet Life and Related Hypotheses in Indonesian Banking Industry." **Viverita** investigates the relation between market power and cost-efficiency (the *quiet life hypothesis*), and the two competing hypotheses of the relationship between market power and efficiency as well as market concentration on profitability (*Structure Conduct Performance* and

Efficient Structure). This is illustrated with an application in the Indonesian banking industry from 2002 to 2011. Further to DEA and to capture the equilibrium dynamic of the Indonesian banking industry, the Lerner index method is used to measure the level of competition. Results of this chapter fail to reject both *Structure Conduct Performance* hypothesis and *Efficient Structure* hypothesis, but disapprove the existence of the *quiet life hypothesis* in the Indonesian banking market.

Internal structure of service organizations is important in service productivity. **Ming-Miin Yu and Li-Hsueh Chen**, in chapter "Internal Structure of Service Organization: From Multi-activity Financial Institutions to Network Structure Hotels," discuss that in recent years, based on characteristics that operational processes of financial institutions and hotels may jointly engage in multiple activities and multiple processes. This chapter is dedicated to describing internal structures of financial institutions and hotels as well as providing relative DEA models and applications. The chapter illustrates that in order to conform to real operational situations, the construction of DEA model should consider and match the internal operational characteristics of decision making units.

As another application **Michael L. Antonio** and **Ma. Socorro P. Calara,** in chapter "Application of DEA in the Electricity Sector: The Case of Meralco Distribution Sectors," present an application of DEA in the electricity sector with the Case of Meralco Distribution Sectors. The chapter seeks to (1) evaluate and compare the technical efficiency performance of Meralco Distribution Sectors using selected Performance-Based Regulation (PBR) indicators and other inputs, (2) determine which Meralco Distribution Sector achieved the highest technical efficiency performance, and (3) identify areas for improvement of each Meralco Distribution Sector. A linear monotone transformation was adapted to make use of an undesirable output in the DEA model. The chapter's findings imply that the management of Meralco or distribution sectors need to formulate strategies and policies that would further improve their performances.

Chapter "Improving Energy Efficiency Using Data Envelopment Analysis: A Case of Walnut Production" is an application of DEA for improving energy efficiency in farms with a case of Walnut Production. **Alireza Khoshroo and Richard Mulwa** discuss that Walnut is one of the most nutritive crops and modern production methods that can require large quantities of energy. Efficient use of these energies is a necessary step towards agricultural sustainability. Hence, this chapter focuses on optimizing energy consumption in walnut production by identifying and reducing excessive use of energy. DEA is used to model efficiency as an explicit function of human labor, machinery, fertilizers-chemicals, and irrigation energies. The result of DEA analysis shows a substantial inefficiency between the Walnut producers in the studied area, with the main difference between efficient and inefficient producers being in the use of chemicals, potash, machinery, and irrigation water.

Chapter "Service Productivity in IT: A Network Efficiency Measure with Application to Communication Systems" focuses on more advanced DEA models such as network efficiency measure with the application to communication systems. **Adeyemi Abel Ajibesin, Neco Ventura, H. Anthony Chan, and Alexandru**

Murgu introduce a network efficiency measure, which is a new kind of thinking for many evaluators in information technology and engineering. Efficiency measure involves going beyond knowledge (real or estimated) of program (nodes, algorithms, networks, etc.) impact and attempting to compare with other programs. In most cases, this knowledge leads to a decision as whether to replace the program with another more effective program. In this chapter, DEA is applied to extend the existing engineering method in computer networks and to evaluate the efficiency of communication networks.

In the same area of IT efficiency, **Geeta Sharma**, in chapter "Efficiency of Software Development Projects: A Case Study on an Information Technology Company in India," applies DEA to evaluate the relative efficiency of software development projects of a leading software company in India. In this chapter, projects are categorized as per their efficiency scores into highly efficient, moderately efficient, and less efficient companies through a process called Tier Analysis. The chapter also includes an improvement path for the projects with low efficiencies. Furthermore, through the application of Kruskal Wallis test, the software development project efficiency is compared with team size to determine whether efficiency varies across various team size categories, i.e., small, medium, large and extra-large.

The rest of this book is on the transport efficiency. **Darold T Barnum, John M Gleason**, **and Matthew G Karlaftis**, in chapter "Protocol for Comprehensive Efficiency Analysis of Multi-Service Metropolitan Transit Agency Operators," present a DEA protocol for analyzing the efficiency of metropolitan transit agencies that oversee multiple types of transportation services. The protocol is illustrated by applying it to United States transit agencies that can serve their cities with four types of subunits: self-operated motorbus, outsourced motorbus, self-operated demand-responsive, and outsourced demand-responsive. Using DEA models adapted for non-substitutable inputs and outputs, scores estimated for a focus agency include: (1) technical efficiency of the focus agency as a whole, (2) technical efficiency of each of the focus agency's subunit types when each subunit is compared only to others of the same type, (3) allocation efficiency of the focus agency in apportioning resources among its subunits, and (4) the effect of each subunit's technical efficiency on its parent agency's technical efficiency. Finally, a mathematical programming algorithm is illustrated that allocates the focus agency's resources to its subunits with the objective of decreasing the cost of transit in an urban area while holding total ridership constant. The protocol thereby is a comprehensive analysis and synthesis of a focus transit agency's efficiency in providing services to its metropolitan area.

Measuring the sustainability of air navigation services is subject of the chapter "Measuring the Sustainability of Air Navigation Services." **Vladimir Grigorov and Paula Rachel Mark** discuss that service productivity is synonymous with the organizational sustainability. It has applications that are broader than conserving the environment via agroindustrial innovation. The domain of Air Navigation Services is a classic example of a service industry, the sustainability of which can be determined using its organizational efficiency and effectiveness. It is a challenge to measure these organizational factors in this profession, because of insufficient data and the effect of random events such as inclement weather that cannot be quantified. A DEA caters for these restrictions and is thus an appropriate tool for determining the sustainability of Air Navigation Service Providers. The DEA results highlight the need for urgent attention to the organizational structure of Air Navigation Services and the reallocation of resources that will improve sustainability.

Sreekanth Mallikarjun, Herbert F. Lewis, and Thomas R. Sexton in chapter "Measuring and Managing the Productivity of U.S. Public Transit Systems: An Unoriented Network DEA" explain that the U.S. governments at all levels face budget shortfalls, and consequently public transit systems in the United States must compete with other public services for financial support. In order to depend less on public funding, it is critical that public transit systems focus on their operational performance and identify any sources of inefficiency. In this chapter, they present an unoriented network DEA methodology that measures a public transit system's performance relative to its peer systems, compares its performance to an appropriate efficient benchmark system, and identifies the sources of its inefficiency.

In the same area of public transport, **Thomas R. Sexton, Allan J. Jones, Andy Forsyth, and Herbert F. Lewis**, in chapter "Using DEA to Improve the Efficiency of Pupil Transportation," provide an example of use of DEA in Washington State that like many other states spends hundreds of millions of dollars annually to support the transportation of pupils to and from school. As with other state-funded activities, inefficiency increases costs and saps resources away from other critical state functions such as public and higher education, health care, transportation, and many others. In 2006, the state undertook a project to revise its pupil transportation funding formula and encourage its school districts to operate more efficiently. Together with Management Partnership Services, Inc., the state developed a DEA-based efficiency measurement system that it now uses to identify inefficient pupil transportation systems for management intervention. The system has identified potential first-year savings of roughly $33 million, with recurrent annual savings of at least $13 million.

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We hope the readers will share our excitement with this important scientific contribution to the body of knowledge in use of Data Envelopment Analysis to Managing Service Productivity.

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Incorporating Intra- and Inter-Input/Output Weight Restrictions in Piecewise Linear DEA: An Application to the Assessment of the Research Activity in Higher Education

Dimitris Sotiros, Yannis G. Smirlis, and Dimitris K. Despotis

Abstract Standard Data Envelopment Analysis models view all input/output factors as linear value functions. Piecewise Linear Data Envelopment Analysis (PL-DEA) is a generalization of the DEA methodology which incorporates piecewise linear functions of factors to handle situations that do not have a linear impact on efficiency, as they may exhibit either diminishing or increasing marginal values. In this chapter we extend PL-DEA to Value-based PL-DEA that incorporates value judgments and allows common treatment for intra- and inter-input/output weight restrictions. Value-based PL-DEA further enables a better expression of individual preferences, enhances the model with the fully units invariance property and also resolves the discontinuity issue that exist in the original PL-DEA model. Value based PL-DEA is illustrated by an application to assess the quality and extent of research work in higher education.

Keywords Data Envelopment Analysis • PL-DEA • University assessment • Efficiency • Managing Service Productivity

1 Introduction

Although the flexibility privileged to the evaluated unit in selecting its own weights is one of the major advantages of DEA in locating inefficiencies, the weights assigned to the inputs and outputs may not be necessarily in line with the individual preferences of a decision maker in his efficiency assessment project. To address this issue, various methods to incorporate value judgments in DEA efficiency assessments have been arisen. The necessity to drive the weights assigned to the factors originates from a variety of reasons, such as to improve the discrimination power of

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DEA, to restrain the diversity of the weights assigned to the same factor by different DMUs and to incorporate individual preferences and trade-offs over the inputs and outputs.

There are two broad classes of methods to incorporate value judgments in DEA. The one is based on an explicit restriction of the weight space by imposing either direct constraints on the weight variables (Cook et al. 1991; Dyson and Thanassoulis 1988; Thompson et al. 1986, 1990) or constraints on the virtual inputs and outputs, i.e. the input/output measures multiplied by the weights (Wong and Beasley 1990). The other concerns the alteration of the data space, either by altering the data set itself, such as the Cone–Ratio approach of Charnes et al. (1989), or by introducing fictitious DMUs (Golany and Roll 1994; Halme et al. 1999; Podinovski 2004; Thanassoulis and Allen 1998). The reader is referred to Thanassoulis et al. (2004) for a comprehensive review and interpretations of the various methods.

The recently introduced piecewise linear DEA (PL-DEA) deals with problems where there are inputs and/or outputs that do not have a linear impact on efficiency, as they may exhibit either diminishing or increasing marginal values. Concavity or convexity of the value functions in PL-DEA is driven by weight restrictions of assurance region type, applied to an augmented data set. When value judgments are complex, the mixture of weight restrictions for the input/output factors individually and the data segments within these factors may not be straightforward.

In this chapter we first review the previous work in PL-DEA. Then, building on this, we reformulate the PL-DEA model in a value-based manner that allows common treatment for intra- and inter-input/output weight restrictions. The reformulation constitutes of data normalization and the introduction of new variables representing the worth of the maximum observed level of input/output. The weight restrictions are then translated in terms of these new variables. This approach enables a better expression of individual preferences, enhances the model with the fully unit invariance property and also deals with discontinuity problems that exist in the original PL-DEA model.

The rest of the chapter is structured as follows: Section 2 reviews the Piecewise Linear Data Envelopment Analysis (PL-DEA) and presents its basic formulations. Section 3 extends the PL-DEA to the value based PL-DEA that better express the preferences of decision makers and solves shortcomings of the previous approach. Section 4 illustrates the above mentioned techniques by presenting an application to assess the quality and extent of research work in higher education. The chapter ends with concluding remarks and conclusions.

2 Piecewise Linear Data Envelopment Analysis: A Review

2.1 DEA, Linearity of Virtual Outputs and Inputs

Assume n units, each using m inputs to produce s outputs. We denote by y_{rj} the level of the output r ($r = 1,\ldots,s$) produced by unit j ($j = 1,\ldots,n$) and by x_{ij} the level of the input i ($i = 1,\ldots, m$) consumed by the unit j. The multiplier form of the output-oriented VRS DEA model (Banker et al. 1984) for evaluating the relative efficiency of the unit j_0 is as follows:

$$\begin{aligned}
&\min\ h_{j_0} = \sum_{i=1}^{m} v_i x_{ij_0} - w_0 \\
&s.t. \\
&\sum_{r=1}^{s} u_r y_{rj_0} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + w_0 \le 0, j = 1, \ldots, n \\
&u_r \ge 0 \ \ (r = 1, \ldots, s) \\
&v_i \ge 0 \ \ (i = 1, \ldots, m) \\
&w_0 \in \Re
\end{aligned} \tag{1}$$

If $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$ and $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ denote respectively the vectors of outputs and inputs for unit j, the summations in the constraints of model (1) $\sum_{r=1}^{s} u_r y_{rj} = U(Y_j)$ and $\sum_{i=1}^{m} v_i x_{ij} = V(X_j)$ represent the total virtual output and input respectively for unit j. The value functions $U(Y_j)$ and $V(X_j)$, rewritten in the additive form in terms of the partial functions $U_r(y_{rj}) = u_r y_{rj}$ and $V_i(x_{ij}) = v_i x_{ij}$

$$U(Y_j) = U_1(y_{1j}) + U_2(y_{2j}) + \ldots + U_s(y_{sj}) \tag{2}$$

$$V(X_j) = V_1(x_{1j}) + V_2(x_{2j}) + \ldots + V_m(x_{mj}) \tag{3}$$

are linear functions of the weights. This linearity imposes a linear impact on the efficiency as the values x_{ij} and y_{rj} vary within the ranges of inputs and outputs. However, this situation cannot reflect the reality in a number of applications in which the value of an input/output exhibits diminishing marginal value (DMV) or increasing marginal value (IMV) beyond certain thresholds. In such cases, the value functions $U(Y_j)$ and $V(X_j)$ have a piecewise linear form which the original DEA model is unable to handle.

2.2 *Piecewise Linear DEA*

Piecewise linear DEA (PL-DEA) is an extension of DEA dealing with cases where the partial value functions have a piecewise linear form.

PL-DEA was first introduced by Cook and Zhu (2009) to handle DMV/IMV in certain outputs in an application that measures the efficiency of maintenance patrols in the province of Ontario, Canada. Cook et al. (2009) further extended PL-DEA in the additive model for inputs with diminishing values. Despotis et al. (2010) provided a general CCR modeling approach for the efficiency assessment in the presence of nonlinear virtual inputs and outputs in terms of assurance region constraints to implement concave output and convex input value functions. For the illustration, they revisit previous work dealing with the assessment of the human development index on the light of DEA. Furthermore, Lofti et al. (2010) noticed that the PL-DEA model fails to produce acceptable targets so they revised the PL-DEA by proposing a two stage CCR modeling that handles the problem of setting the targets of the units precisely. PL-DEA has been also adapted to interval DEA (Smirlis and Despotis 2013), i.e. to cases where the input/output data are only known to lie within intervals with given bounds. The authors defined appropriate interval segmentations to implement the piecewise linear forms in conjunction with the interval bounds of the input/output data.

PL-DEA has been also used as the background technique in Smirlis and Despotis (2012) to handle extreme observations (those that exhibit irregularly high values in some outputs and/or low values in some inputs) in DEA, instead of removing them from the analysis. Their modeling approach assumed that the contribution of output dimensions that show extreme values, to the efficiency score diminishes as the output increases beyond a pre-specified level. Using such pre-specified threshold levels as breakpoints, they applied the PL-DEA concept of diminishing returns to implement piecewise concave value functions.

PL-DEA is formulated as follows. Focusing on outputs, the linearity of the partial value functions $U_r(y_{rj}) = u_r y_{rj}$ is relaxed and assumed to be piecewise linear. Let $[l_r, h_r]$ be the range of output r over the entire set of DMUs (i.e. $l_r = \min_j\{y_{rj}\}$ and $h_r = \max_j\{y_{rj}\}$). For each output $r = 1,\ldots,s$, the interval $[l_r, h_r]$ is segmentized by considering a number α_r of breakpoints $b_r^1, b_r^2, \ldots, b_r^k$, $b_r^{k+1}, \ldots, b_r^{\alpha_r}$ with $b_r^1 = l_r$ and $b_r^{\alpha_r} = h_r$. Then for $y_{rj} > l_r$ there is exactly one k_j such that $y_{rj} \in \left(b_r^{k_j}, b_r^{k_j+1}\right]$ and, in terms of the breakpoints, y_{rj} can be decomposed as follows:

$$y_{rj} = b_r^1 + \left(b_r^2 - b_r^1\right) + \left(b_r^3 - b_r^2\right) + \ldots + \left(b_r^{k_j} - b_r^{k_j-1}\right) + \left(y_{rj} - b_r^{k_j}\right) \quad (4)$$

Instead of considering a single weight variable throughout the interval $[l_r, h_r]$, a different weight variable is assigned for each subinterval $(b_r^k, b_r^{k+1}]$. Let u_{r1} denote the weight assigned to b_r^1, u_{r2} the weight assigned to the first subinterval $(b_r^1, b_r^2]$ and u_{rk} the weights assigned to the subsequent subintervals $(b_r^k, b_r^{k+1}]$ for $k = 2,\ldots,\alpha_r - 1$ and $r = 1,\ldots,s$. Then setting

$$\begin{aligned} &\gamma_{r1}^j = b_r^1, \gamma_{r2}^j = b_r^2 - b_r^1, \gamma_{r3}^j = b_r^3 - b_r^2, \ldots, \gamma_{rk_j}^j = b_r^{k_j} - b_r^{k_j-1}, \\ &\gamma_{rk_j+1}^j = y_{rj} - b_r^{k_j}, \gamma_{rk_j+2}^j = 0, \ldots, \gamma_{r\alpha_r}^j = 0 \end{aligned} \quad (5)$$

the value $U_r(y_{rj})$ for any $y_{rj} \in \left(b_r^{k_j}, b_r^{k_j+1}\right]$ is written:

$$U_r\left(y_{rj}\right) = \gamma_{r1}^j u_{r1} + \gamma_{r2}^j u_{r2} + \ldots + \gamma_{r,k_j}^j u_{r,k_j} + \gamma_{r,k_j+1}^j u_{r,k_j+1} + \gamma_{r,k_j+2}^j u_{r,k_j+2} + \ldots + \gamma_{r,a_r}^j u_{r,a_r} \tag{6}$$

Writing (6) for every output $r = 1, \ldots, s$ and summing over r we get the virtual output $U(Y_j)$ for every unit j as a function of $u = \left(u_{11}, \ldots, u_{1,a_1}, \ldots, u_{r1}, \ldots, u_{r,a_r}, \ldots, u_{s1}, \ldots, u_{s,a_s}\right)$:

$$U\left(Y_j\right) = \sum_{r=1}^{s} \sum_{k=1}^{a_r} \gamma_{rk}^j u_{rk} \tag{7}$$

In (7), the partial value functions are considered nonlinear for all outputs. However, this is not the case in general. The nonlinearity assumption may be applicable or desirable for particular outputs only. For simplicity, let us call these outputs *non-linear*, and let us call *linear* the rest of them. Without loss of generality, we assume that the outputs are ordered in a manner that the first d ($d < s$) are linear and the rest of them (i.e. for $r = d + 1, \ldots, s$) non-linear. Then (7) takes the following form:

$$U\left(Y_j\right) = \sum_{r=1}^{d} y_{rj} u_r + \sum_{r=d+1}^{s} \sum_{k=1}^{a_r} \gamma_{rk}^j u_{rk} \tag{8}$$

The virtual inputs can be similarly modeled in a piecewise linear fashion. Indeed, the overall value of the input vector $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ of unit j is given by the additive function $V(X_j) = V_1(x_{1j}) + V_2(x_{2j}) + \ldots + V_m(x_{mj})$. As in the case of outputs, particular inputs may be assumed having non-linear partial value functions. Accordingly, let the first t inputs ($t < m$) be linear and the rest of them non-linear. Then for $i = t + 1, \ldots, m$ and a set of breakpoints $b_i^1, b_i^2, \ldots, b_i^k, b_i^{k+1}, \ldots, b_i^{a_i}$, the input value $x_{ij} \in \left(b_i^{k_j}, b_i^{k_j+1}\right]$ of unit j is formulated as follows:

$$x_{ij} = b_i^1 + \left(b_i^2 - b_i^1\right) + \left(b_i^3 - b_i^2\right) + \ldots + \left(b_i^{k_j} - b_i^{k_j-1}\right) + \left(x_{ij} - b_i^{k_j}\right) \tag{9}$$

$$\delta_{i1}^j = b_i^1, \delta_{i2}^j = b_i^2 - b_i^1, \ldots, \delta_{ik_j}^j = b_i^{k_j} - b_i^{k_j-1}, \delta_{ik_j+1}^j = x_{ij} - b_i^{k_j}, \delta_{ik_j+2}^j = 0, \ldots, \delta_{i\alpha_i}^j = 0 \tag{10}$$

$$V_i\left(x_{ij}\right) = \delta_{i1}^j v_{i1} + \delta_{i2}^j v_{i2} + \ldots + \delta_{ik_j}^j v_{ik_j} + \delta_{ik_j+1}^j v_{ik_j+1} + \delta_{ik_j+2}^j v_{i,k_j+2} + \ldots + \delta_{ia_i}^j v_{ia_i} \tag{11}$$

The overall value $V(X_j)$ for unit j is given by

$$V(X_j) = \sum_{i=1}^{t} x_{ij} v_i + \sum_{i=t+1}^{m} \sum_{k=1}^{a_i} \delta_{ik}^{j} v_{ik} \tag{12}$$

The definition of the auxiliary variables $\gamma_{rk_j}^{j}$, $\delta_{ik_j}^{j}$ in terms of the breakpoints decompose each one of the non-linear outputs y_{rj} and inputs x_{ij} into auxiliary linear parts thus the original data set is transformed to an augmented data set. This transformation allows performing the efficiency assessments without drawing away from the grounds of the standard DEA methodology.

The following model (13) is a piecewise linear output-oriented VRS DEA model.

$$\begin{aligned}
&\min\ h_{j_0} = V(X_{j_0}) - w_0 \\
&s.t. \\
&U(Y_{j_0}) = 1 \\
&U(Y_j) - V(X_j) + w_0 \le 0, \quad j = 1, \ldots, n \\
&V(X_j) = \sum_{i=1}^{t} x_{ij} v_i + \sum_{i=t+1}^{m} \sum_{k=1}^{a_i} \delta_{ik}^{j} v_{ik} \\
&U(Y_j) = \sum_{r=1}^{d} y_{rj} u_r + \sum_{r=d+1}^{s} \sum_{k=1}^{a_r} \gamma_{rk}^{j} u_{rk} \\
&u_r \ge 0\ (r = 1, \ldots, d) \\
&v_i \ge 0\ (i = 1, \ldots, t) \\
&u_{rk} \ge 0\ (r = d+1, \ldots, s; k = 1, \ldots, a_r) \\
&v_{ik} \ge 0\ (i = t+1, \ldots, m; k = 1, \ldots, a_i) \\
&w_0 \in \Re; u_r, v_i, u_{rk}, v_{ik} \in \Omega
\end{aligned} \tag{13}$$

In model (13), Ω is the set of restrictions on the weights that reflect the preferences of the analyst.

2.3 Assurance Region Model for Concave Output and Convex Input Value Functions

Each partial value function $U_r, r = d+1,.., s$ is a non-decreasing function over the range of output y_r. However, to provide U_r with additional properties, such as concavity, it is necessary to restrict the weights assigned to the successive linear parts of U_r. So, to represent the situation where for a particular output its marginal

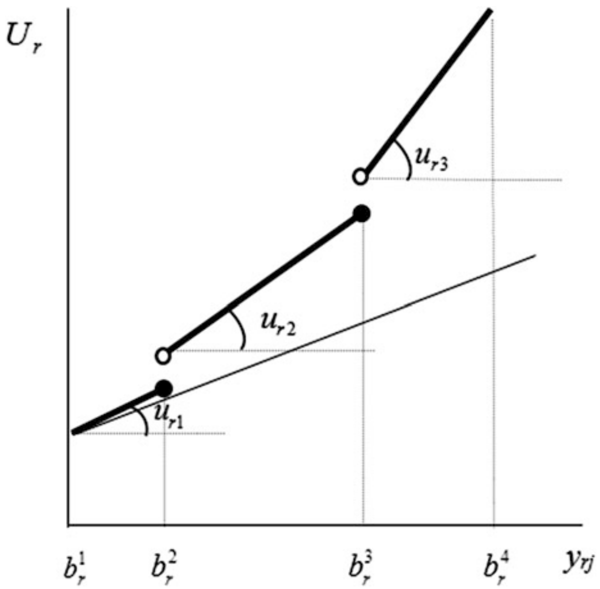


Fig. 1 Convex form

value diminishes as the output increases, the partial value function U_r is modeled as in (6) by adding the following homogeneous restrictions on the weights:

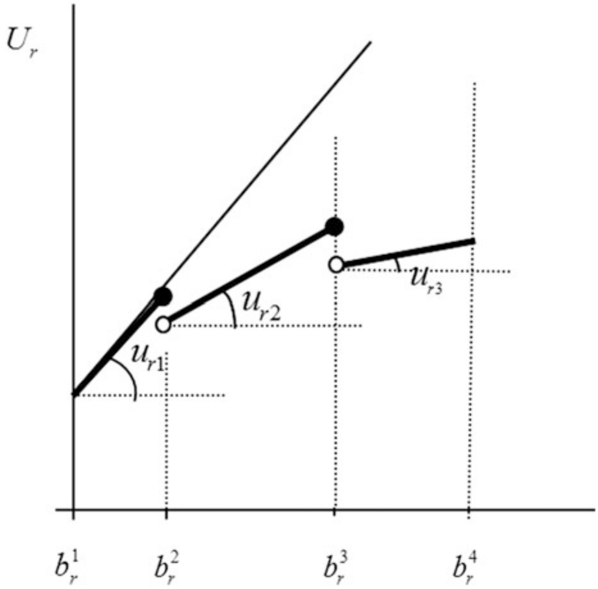
$$c_{rk} \leq \frac{u_{rk}}{u_{rk+1}} \leq z_{rk}, k = 1, .., a_r - 1 \tag{14}$$

where c_{rk}, z_{rk} are real numbers such that $1 < c_{rk} < z_{rk}$.

Figures 1 and 2, present for a typical non-linear output y_r, the convex and concave shape of the non-linear function U_r. Note that the functions U_r in both cases show discontinuity at the breakpoint values $b_i^{k_j}$. This is due to fact that the weights u_{rk} and u_{rk+1} in successive sub-intervals $\left(b_i^{k_j-1}, b_i^{k_j}\right]$ and $\left(b_i^{k_j}, b_i^{k_j+1}\right]$ may be different. This is a defect of PL-DEA which is resolved in the extended version of PL-DEA, presented and discussed in next Sect. 3.

Similarly, the partial value functions of the non-linear inputs $U_i, i = t+1,.., m$ are considered as non-decreasing functions over the range of inputs x_i. In line with the primal element of prospect theory that a value function is concave for gains (outputs) and convex for losses (inputs) and given that the input dimension forms the denominator of the efficiency ratio, we assume further that U_i are convex non-decreasing functions, to represent the situation where for a particular input i its marginal value increases as the input begins to increase. Usage of excess inputs is thus penalized. This situation is modeled by restricting the weights assigned to the successive linear parts of V_i as follows:

Fig. 2 Concave form



$$q_{ik} \leq \frac{v_{ik}}{v_{ik+1}} \leq p_{ik}, k = 1, .., a_i - 1 \tag{15}$$

where q_{ik}, p_{ik} are real numbers such that $0 < q_{ik} < p_{ik} < 1$.

It is important to notice that, depending on a particular application, any other shape of the piecewise value function can be supported by modifying appropriately the constrains (14) and (15).

PL-DEA approach is applicable to cases where the value functions have a piecewise linear form. However, the graphical representation of the value function U_r (see Figs. 1 and 2) reveals a discontinuity due to different weight values u_{rk}, u_{rk+1} on the successive data segments. In addition model (13) is not fully units invariant (Lovell and Pastor 1995) and the interpretation of intra and inter variable restrictions may not be convenient for the decision maker. To deal with these issues we present an effective reformulation of PL-DEA approach.

3 A Value Based PL-DEA Approach

In this section, we introduce a data transformation—variable alteration technique which allows us to reformulate PL-DEA in a more effective way. The new variables represent the worth of the maximum observed level of output/input and the restrictions on the variable space obtain a meaningful interpretation for the decision maker. We present also the translation of weight restrictions to value restrictions and we provide a common treatment for intra- and inter-input/output weight restrictions. This new approach deals also with the discontinuity issue presented in the previous section.

3.1 A Data Transformation: Variable Alteration Technique

Concerning the linear outputs, let $l_r = \min_j \{y_{rj}\}$ and $h_r = \max_j \{y_{rj}\}$ be the minimum and maximum values for output r. The value (virtual output) of any $y_{rj} \in [l_r, h_r]$ is given by: $U_r(y_{rj}) = y_{rj} u_r$

Applying the following transformation:

$$U_r(y_{rj}) = \frac{y_{rj}}{h_r} u_r h_r = \hat{y}_{rj} p_r$$

we get the value of $y_{rj} \in [l_r, h_r]$ as function of the new variable p_r as:

$$U_r(y_{rj}) = \hat{y}_{rj} p_r \tag{16}$$

with

$$\hat{y}_{rj} = \frac{y_{rj}}{h_r}$$

As depicted in Fig. 3, the above transformation alters the weight variable u_r, which represents the slope of the line OA, to the new variable p_r that represents the value of h_r. The coefficient $\hat{y}_{rj}$ is now dimensionless (units free) and the term $\hat{y}_{rj} p_r$ represents the value of the output y_{rj} as a proportion of p_r.

To treat non-linear outputs, we apply to each segment the same transformation introduced for the linear outputs and we get the partial value function of output r for unit j (6) in terms of the new variables $p_{r1}, p_{r2}, \ldots, p_{r,a_r}$ as follows:

$$U_r\left(y_{rj}\right) = \hat{\gamma}^j_{r1} p_{r1} + \hat{\gamma}^j_{r2} p_{r2} \ldots + \hat{\gamma}^j_{r,a_r} p_{r,a_r} = \sum_{k=1}^{a_r} \hat{\gamma}^j_{rk} p_{rk} \tag{17}$$

With $\hat{\gamma}^j_{r1} = \frac{\gamma^j_{r1}}{b^1_r}$ and $\hat{\gamma}^j_{rk} = \frac{\gamma^j_{rk}}{b^k_r - b^{k-1}_r}, k = 2, \ldots., a_r$

It is straightforward from (17) that $U_r(h_r) = p_{r1} + p_{r2} + \ldots + p_{r,a_r}$

Figure 4 depicts a piecewise linear value function for a non-linear output measure Y_r decomposed in two segments. With the above transformations, the weight variables u_{r1}, u_{r2} and u_{r3}, which represent respectively the slopes of the line segments OA, AB and BC, are replaced by the value variables p_{r1}, p_{r2} and p_{r3}, which represent the value increments in the intervals $[0, b^1_r]$, $(b^1_r, b^2_r]$ and $(b^2_r, b^3_r]$ respectively.

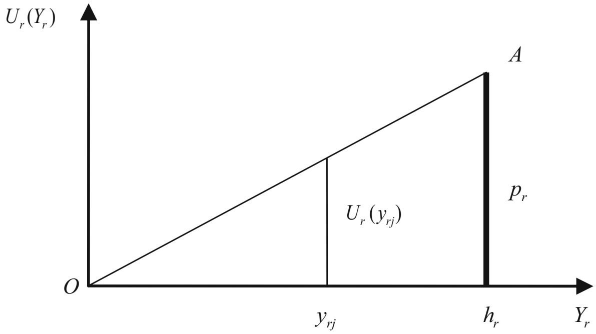


Fig. 3 Value function for a linear output measure Y_r

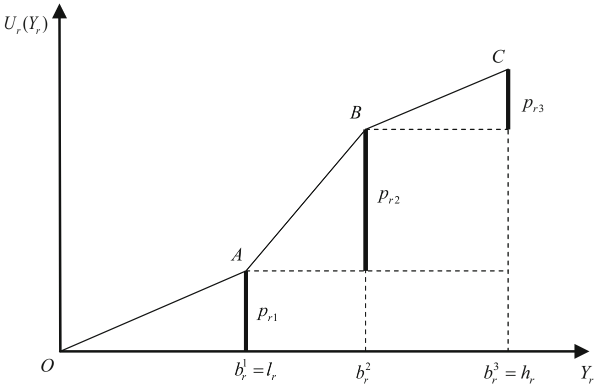


Fig. 4 Value function for a non-linear output measure Y_r

Putting all together, i.e. the value functions of the linear and the non-linear outputs as given in (16) and (17) respectively, we get the value function (total virtual output) for the unit j, as follows:

$$U(Y_j) = \sum_{r=1}^{d} \hat{y}_{rj} p_r + \sum_{r=d+1}^{s} \sum_{k=1}^{a_r} \hat{\gamma}_{rk}^{j} p_{rk} \tag{18}$$

In Eq. (18), the first summation refers to linear outputs, whereas the second summation refers to non-linear outputs.

Concerning the inputs, we treat the value functions in a similar manner. The overall value $V(X_j)$ for unit j is given by

$$V(X_j) = \sum_{i=1}^{t} \hat{x}_{ij} q_i + \sum_{k=t+1}^{a_i} \hat{\delta}_{ik}^{j} q_{ik} \tag{19}$$

Where $\hat{x}_{ij} = \frac{x_{ij}}{h_i}$ and $\hat{\delta}_{i1}^{j} = \frac{\delta_{i1}^{j}}{b_i^1}$, $\hat{\delta}_{rk}^{j} = \frac{\delta_{rk}^{j}}{\alpha_i^k - \alpha_i^{k-1}}, k = 2, \ldots., a_i$

According to Eqs. (18) and (19) the piecewise linear value based DEA model is expressed as follows:

$$
\begin{aligned}
&\min\ h_{j_0} = V\left(X_{j_0}\right) - w_0 \\
&s.t. \\
&U\left(Y_{j_0}\right) = 1 \\
&U\left(Y_j\right) - V\left(X_j\right) + w_0 \leq 0,\ j = 1, \ldots, n \\
&V\left(X_j\right) = \sum_{i=1}^{t} \hat{x}_{ij} q_i + \sum_{k=t+1}^{a_i} \hat{\delta}_{ik}^{\,j} q_{ik} \\
&U\left(Y_j\right) = \sum_{r=1}^{d} \hat{y}_{rj} p_r + \sum_{r=d+1}^{s} \sum_{k=1}^{a_r} \hat{\gamma}_{rk}^{\,j} p_{rk} \\
&p_r \geq 0\ (\ r = 1, \ldots, d) \\
&q_i \geq 0\ (i = 1, \ldots, t) \\
&p_{rk} \geq 0\ (r = d+1, \ldots, s; k = 1, \ldots, a_r) \\
&q_{ik} \geq 0\ (i = t+1, \ldots, m; k = 1, \ldots, a_i) \\
&w_0 \in \Re; p_r, q_i, p_{rk}, q_{ik} \in \hat{\Omega}
\end{aligned}
\tag{20}
$$

In the last constraint of model (20), $\hat{\Omega}$ represents the weight restrictions that reflect the decision maker's preferences translated in terms of worth. This transformation of the variables allows us for a common treatment of intra and inter variable restrictions.

3.2 Inter- and Intra-Variable Restrictions

Concerning the linear output r, the overall value is expressed by the variable p_r. Thus, any trade off among the linear outputs r, $r+1$ can be expressed by the equation $\frac{p_r}{p_{r+1}} \leq \frac{f_r}{f_{r+1}}$ where the constants f_r,f_{r+1} denote the level of trade off.

Analogously, the overall value of the non-linear r is expressed by the summation $\sum_{k=1}^{a_r} p_{rk}$. Thus, any trade off among the non-linear outputs r, $r+1$ can be expressed similarly by the equation $\frac{\sum_{k=1}^{a_r} p_{rk}}{\sum_{k=1}^{a_{r+1}} p_{r+1,k}} \leq \frac{f_r}{f_{r+1}}$.

The implementation for trade off among linear and non-linear outputs or inputs is analogous.

Table 1 Translation of weight restrictions to worth restrictions

–	Stated in terms of weights	Translated in terms of values
Absolute restrictions	$a_r \leq u_r \leq b_r$	$a_r h_r \leq p_r \leq b_r h_r$
Assurance region Type I	$a_{kl} \leq \frac{u_k}{u_l} \leq b_{kl}$	$a_{kl}\frac{h_k}{h_l} \leq \frac{p_k}{p_l} \leq b_{kl}\frac{h_k}{h_l}$
	$w_r u_r + w_k u_k \leq u_l$	$\frac{w_r}{h_r}p_r + \frac{w_k}{h_k}p_k \leq \frac{1}{h_l}p_l$
Assurance region Type II	$a_i v_i \geq u_r$	$\frac{a_i}{h_i}q_i \geq \frac{1}{h_r}p_r$
Restrictions on virtual outputs	$a_r \leq \frac{y_{rj}u_r}{\sum_{r=1}^{s} y_{rj}u_r} \leq b_r$	$a_r \leq \frac{\hat{y}_{rj}p_r}{\sum_{r=1}^{s} \hat{y}_{rj}p_r} \leq b_r$

Intra-variable restrictions provide the piecewise linear value functions for non-linear outputs with additional properties which reflect the decision maker's preferences (e.g. concave or convex value functions). Restrictions (14) can be translated to value restrictions as follows:

$$c_{rk} \leq \frac{u_{rk}}{u_{r,k+1}} \leq z_{rk} \Leftrightarrow \frac{b_r^k - b_r^{k-1}}{b_r^{k+1} - b_r^k}c_{rk} \leq \frac{\left(b_r^k - b_r^{k-1}\right)u_{rk}}{\left(b_r^{k+1} - b_r^k\right)u_{rk+1}} \leq \frac{b_r^k - b_r^{k-1}}{b_r^{k+1} - b_r^k}z_{rk} \Leftrightarrow$$
$$\Leftrightarrow \frac{b_r^k - b_r^{k-1}}{b_r^{k+1} - b_r^k}c_{rk} \leq \frac{p_{rk}}{p_{rk+1}} \leq \frac{b_r^k - b_r^{k-1}}{b_r^{k+1} - b_r^k}z_{rk}, k = 2, .., a_r - 1$$

In Table 1, are presented the translation of weight restrictions to value restrictions and vice versa.

3.3 Discontinuity Issue

As mentioned in the previous section, the augmentation of the dataset for non-linear outputs/inputs and the assignment of a distinct weight variable to each segment, may lead to a discontinuity issue in the breakpoints for the value functions. However, applying the data transformation-variable alteration technique, introduced in this section, the new variables represent the increment of worth in each sub-segment. Thus, this technique enables us also to overcome this issue and the value functions acquire a continuous shape (see Fig. 4).

4 Evaluation of the Research Activity of Academic Staff: A PL-DEA Approach

We apply the PL-DEA value-based model described in the previous section, to assess the efficiency of the research activity of academic staff in higher education. The aim is to encompass in the assessments both the volume as well as the quality of the research work. This is made by rewarding the researchers with qualitative research records (i.e. publications in highly ranked journals with significant number of citations) and, contrary, penalizing those that exhibit extensive publications in unranked journals with insignificant contribution.

To this end, we use an anonymous dataset of 112 researchers, faculty members with Business and Economics of Greek Universities. The data were drawn from Scopus, Google Scholar, university staff records and their personal Curriculum Vitaes. The input and output factors for this assessment are as follows (Table 2).

Table 3 presents the descriptive statistics for the collected input-output data.

Additional information about the distribution of the values of certain input/output factors is provided in Figs. 5, 6, 7 and 8. In order to facilitate the assessment of the quality and extent of the research work, certain intra- and inter-variable constraints are introduced. To put emphasis on the quality of the research outcome, the factors *Publications in A+*, *A journals*, *Publications in B*, *C journals*, *Publications in unranked journals* and *Number of Citations* are considered as non-linear (the rest of the factors are assumed linear). Especially for the output factor *Publications in A+*, *A journals*, convex value function is assumed so as to reward those showing relatively high volume of quality publications. A single breakpoint is set to $b_1^2 = 8$ (Fig. 5), while the convexity shape of the value function is derived by the condition $\frac{u_{12}}{u_{13}} \leq \frac{1}{2}$. Similar arrangement is made for the factors *Publications in B*, *C journals* and *Number of Citations* for which the corresponding breakpoints are set to $b_2^2 = 18$ and $b_6^2 = 200$, respectively and the convexity conditions are $\frac{u_{22}}{u_{23}} \leq 1$ and $\frac{u_{62}}{u_{63}} \leq \frac{1}{2}$. Contrarily, a concave value function is assumed for the factor *Publications in unranked journals* so as to penalize those researchers showing a relative large number of publications in non-quality journals. For that factor the breakpoint is set to $b_3^2 = 18$ and the concave shape of the value function is derived by the condition $\frac{u_{32}}{u_{33}} \geq 2$.

In addition to the intra-variable restrictions that form the convex and concave shape of the value function, inter-variable restrictions are employed to define certain priorities among the output factors that describe the research outcome. These conditions are shown in the following Table 4.

The following table presents the intra-variable and inter-variable restrictions in terms of the new variables representing worth (Table 5).

By applying the value-based PL-DEA model (20) to the data set and by comparing the resulting efficiency scores with the original DEA model (1), we observe a significant reduction of the efficiency scores and consequently of the number of efficient researchers as indicated in the following Table 6.

Table 2 Input and outputs

Input	–
I. Years of research activity	Time (years) since the first publication
Outputs	–
O_1.Publications in A+, A journals	Number of publications in highly ranked journals (rank A+ or A) according to the ERA 2010[a]
O_2. Publications in B, C journals	Number of publications in medium ranked journals (rank B, C) according to the ERA 2010[a]
O_3. Publications in unranked journals	Number of publications in unranked journals
O_4. Conference proceedings	Number of publications in proceedings of national and international conferences
O_5. Research projects	Number of research projects that the researcher participated
O_6. Citations	Number of citations (excluding self-citations)

[a]Excellence in Research for Australia (ERA) 2010 journal classification system—http://www.arc.gov.au

Table 3 Descriptive statistics for input and outputs

Variable	Mean	StDev	Minimum	Median	Maximum
I	17.78	6.72	5.00	17.00	30.00
O_1	6.14	4.67	0.00	5.000	19.00
O_2	11.74	7.51	0.00	10.00	29.00
O_3	14.88	10.93	0.00	13.00	47.00
O_4	34.98	21.97	1.03	32.50	104.00
O_5	6.35	4.58	1.00	5.000	15.00
O_6	56.23	78.21	0.00	23.00	350.00

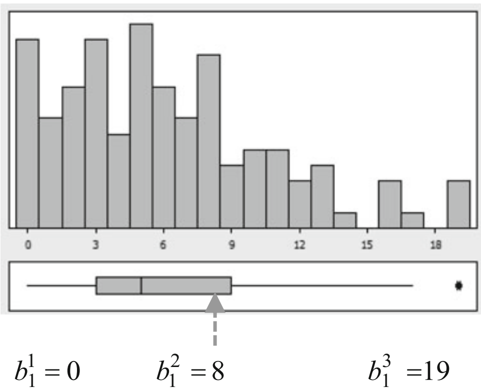


Fig. 5 Publications in A+, A journals

To further illustrate the effectiveness of value based PL-DEA approach in capturing the quality and extent of the research work, we focus and analyze as follows a number of researchers selected form the data set.

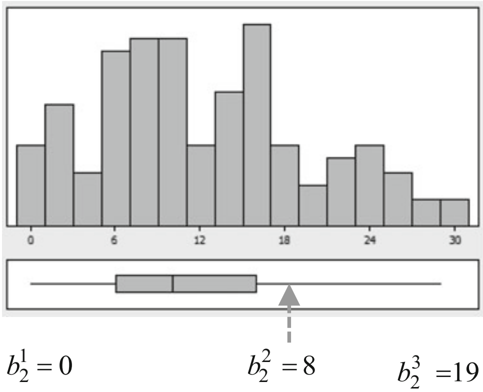


Fig. 6 Publications in B, C journals

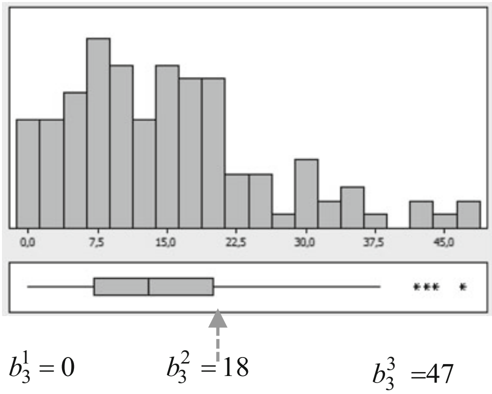


Fig. 7 Publications in unranked journals

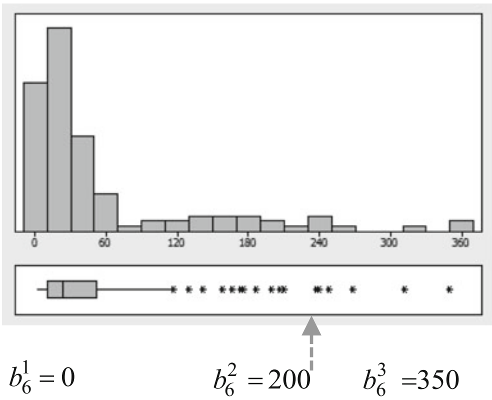


Fig. 8 Number of Citations

Table 4 Inter-variable weight restrictions

Weight of publications in A+, A journals $\geq$ 2*weight of publications in B, C journals
Weight of publications in B, C journals $\geq$ 3*weight of publications in unranked journals
Weight of publications in B, C journals $\geq$ 3*weight of conference proceedings

Table 5 Restrictions translated in terms of worth

Intra-variable restrictions	Inter-variable restrictions
$\frac{p_{12}}{p_{13}} \leq \frac{1}{2} \frac{(b_1^2 - b_1^1)}{(b_1^3 - b_1^2)}$	$p_{11} + p_{12} + p_{13} \geq 2 * (p_{21} + p_{22} + p_{23})$ $p_{21} + p_{22} + p_{23} \geq 3 * (p_{31} + p_{32} + p_{33})$
$\frac{p_{22}}{p_{23}} \leq \frac{(b_2^2 - b_2^1)}{(b_2^3 - b_2^2)}$	$p_{21} + p_{22} + p_{23} \geq 3 * (p_{41} + p_{42} + p_{43})$
$\frac{p_{62}}{p_{63}} \leq \frac{1}{2} \frac{(b_6^2 - b_6^1)}{(b_6^3 - b_6^2)}$	
$\frac{p_{32}}{p_{33}} \geq 2 \frac{(b_3^2 - b_3^1)}{(b_3^3 - b_3^2)}$	

Table 6 Number of efficient researchers, average efficiency score

	DEA (model (1))	Value based PL-DEA (model (20))
Number of efficient researchers	27	17
Average efficient score	0.641	0.425

Table 7 Number of efficient researchers, average efficiency score

Factor	Case #1	Case #2	Case #3
I. Years of research activity	28	9	6
O_1. Publications in A+, A journals	19	8	1
O_2. Publications in B, C journals	28	20	0
O_3. Publications in unranked journals	34	31	7
O_4. Conference proceedings	91	15	14
O_5. Research projects	15	5	1
O_6. Citations	268	35	2
Value-based PL-DEA (model (20))	1	1	0.389
DEA (model (1))	1	1	0.823

(i) A subset of ten poor performing researchers satisfying the condition *Years of Research Activity* $\geq$ *20*, *Publications in A+, A journals* $\leq$ *4* and *Publications in unranked journals* $\geq$ *17* has been identified. The average values of efficiency score in cases of standard DEA model (1) and Value-Based PL-DEA model (20) are 0.563 and 0.199 respectively, indicating a significant reduction of their efficiency. This result is strengthened from the fact that the number of efficient researchers when applying Value-Based PL-DEA was eliminated while in case of standard DEA the number of efficient researchers was two.

(ii) Three researchers #1, #2 and #3 are selected as typical cases representing a well performing researcher with adequate years of research activity (case #1) and two young researchers with significant and a poor activity ((cases #2 and #3 respectively). Their performance and efficiency scores are presented in Table 7.

Form Table 7 derives that the quality and extent of research activity in cases #1 and #2 has been rewarded (efficiency scores = 1) and the poor performance in case #3 has been further penalized by the Value-based PL-DEA model (20).

5 Conclusions

In this chapter, we introduced a data transformation—variable alteration technique that allows reformulating PL-DEA in a more effective way. The new variables acquire a meaningful interpretation for the decision maker and the model is enhanced with the fully units invariance property as well as it deals with the discontinuity issue observed in the original PL-DEA. To highlight the new approach, an application for the efficiency assessment of researchers in higher education is presented.

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