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Efficiency assessment in two-stage processes: A novel network DEA approach

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Abstract

A two-stage production process assumes that the first stage transforms external inputs to a number of intermediate measures, which then are used as inputs to the second stage that produces the final outputs. The fundamental approaches to two-stage DEA are the multiplicative and the additive efficiency-decomposition approaches. Both they assume a series relationship between the two stages but they differ in the definition of the overall system efficiency as well as in the way they conceptualize the decomposition of the overall efficiency to the efficiencies of the individual stages. In this paper, we present a novel approach to estimate unique and unbiased efficiency scores for the individual stages, which are then composed to obtain the efficiency of the overall system, by selecting the aggregation method a posteriori. The results derived from our approach are compared with those obtained by the aforementioned basic methods on experimental data as well as on test data drawn from the literature.

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Keywords: Data Envelopment analysis; Network DEA; Two-stage processes.

1. Introduction

Data envelopment analysis (DEA) is the dominating technique for evaluating the relative performance of decision making units (DMUs) that use multiple inputs to produce multiple outputs. The two basic DEA models, namely the CCR model[1] and the BCC model[2], have become standards in the literature of performance measurement under the assumptions of constant and variable returns-to-scale respectively. Conventional DEA models treat the DMUs as single stage production processes that transform inputs to outputs. However, a significant number of studies has focused on assessing efficiency in multistage production processes, where outputs from some stages, characterized as intermediate products, are used either as inputs to the other stages or as external outputs of the production process or both. Färe and Grosskopf[3] were among the first to deal with

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efficiency assessments in such processes, represented as network activity analysis models. Castelli et al.[4] provide a comprehensive categorized overview of models and methods developed for different multi-stage production architectures. In this paper, we focus on the typical architecture of a two-stage production process, which assumes that the external inputs entering the first stage of the process are transformed to a number of intermediate measures that are then used as inputs to the second stage to produce the final outputs. Kao and Hwang[5] introduced an approach by taking into account a series relationship of the two stages and developed a model to estimate the overall efficiency of the production process as geometric average of the efficiencies of the two individual stages. To link the two stages, they assumed that the weighting scheme used for the intermediate measures is common for both stages. Chen et al.[6] introduced the additive efficiency decomposition in two-stage processes. They derive the overall efficiency of the production process as a weighted arithmetic average of the efficiencies of the individual stages, where the weights of the two stages derive endogenously by the optimization process and they are different for each evaluated DMU. An issue investigated further in the literature is the derivation of the efficient frontier in two-stage network DEA. Chen et al[7] pointed out that adjusting the inputs and the outputs by the efficiency scores is not sufficient to yield a frontier projection, when the additive decomposition model is assumed. They developed instead, a model for deriving the efficient frontier within the Kao and Hwang[5] multiplicative framework. The inability of the two-stage DEA models to locate correctly the efficient frontier, as it is the case with standard DEA, is further examined in Chen et al.[8], where different methods to overcome this deficiency are reviewed.

In this paper we provide a short review of the additive and the multiplicative efficiency-decomposition methods to discuss their shortcomings. Then, based on a reverse perspective on how to obtain and aggregate the stage efficiencies, that of the composition as opposed to the decomposition, we develop a novel approach to two-stage DEA that overcomes the deficiencies of the aforementioned decomposition methods.

The paper unfolds as follows. In section two, we provide a review of the additive and the multiplicative efficiency decomposition approaches and we spot their inherent limitations and shortcomings. In section three, we introduce a novel approach to assess the individual efficiencies of the two stages and the overall efficiency of the two-stage process, which effectively overcomes the shortcomings of the additive and the multiplicative decomposition methods. The methods are compared on data drawn from the literature. Conclusions are given in section four.

2. The multiplicative and the additive decomposition methods

Consider the generic case where each DMU transforms some external inputs $\boldsymbol{X}$ to final outputs $\boldsymbol{Y}$ via the intermediate measures $\boldsymbol{Z}$ with a two-stage process, as depicted in Fig. 1.

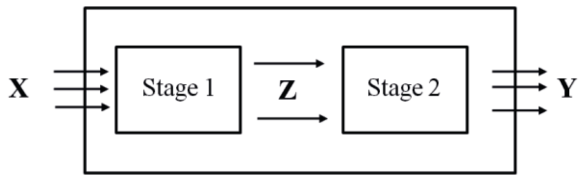

Fig. 1.The architecture of a generic two-stage process

Assume n DMUs (j=1,...,n), each using m external inputs x_{ij}, i=1,...,m in the first stage to produce q outputs z_{pj}, p=1,...,q from that stage. The outputs obtained from the first stage are then used exclusively as inputs to the second stage to produce s final outputs y_{rj}, r=1,...,s. Let us introduce the following basic notation:

Notation

$j \in J = \{1,...,n\}$	The index set of the n DMUs
$j_0 \in J$	Denotes the evaluated DMU
$\boldsymbol{X}_j = (x_{ij}, i = 1,...,m)$	The vector of external inputs used by DMU_j , $j \in \boldsymbol{J}$
$\boldsymbol{Z}_j = (z_{pj}, p = 1,...,q)$	The vector of intermediate measures for DMU_j , $j \in \boldsymbol{J}$
$\boldsymbol{Y}_j = (y_{rj}, r = 1,...,s)$	The vector of final outputs produced by DMU_j , $j \in \boldsymbol{J}$
$\boldsymbol{v} = (v_1,...,v_m)$	The vector of variable weights associated with the external inputs
$\boldsymbol{w} = (w_1,...,w_q)$	The vector of variable weights associated with the intermediate measures
$\boldsymbol{u} = (u_1,...,u_s)$	The vector of variable weights associated with the final outputs
e_j^o	The overall efficiency of DMU_j , $j \in \boldsymbol{J}$
e_j^1	The efficiency of the first stage for DMU_j , $j \in \boldsymbol{J}$
e_j^1	The efficiency of the first stage for DMU_j, $j \in \boldsymbol{J}$

2.1. The multiplicative method

In the multiplicative method[5], the overall efficiency and the stage efficiencies of the DMU_j are defined as follows:

$$e_j^o = \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{v}\boldsymbol{X}_j}, e_j^1 = \frac{\boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j}, e_j^2 = \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{w}\boldsymbol{Z}_j} \tag{1}$$

whereas the decomposition model used is

$$e_j^o = \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{v}\boldsymbol{X}_j} = \frac{\boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j} \cdot \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{w}\boldsymbol{Z}_j} = e_j^1 \cdot e_j^2 \tag{2}$$

i.e. the overall efficiency is the *square geometric average* of the stage efficiencies. Given the above definitions, the linear model below assesses the overall efficiency of the evaluated unit j_0:

$$
\begin{aligned}
& e_{j_0}^{o} = \max \boldsymbol{u}\boldsymbol{Y}_{j_0} \\
& s.t. \\
& \boldsymbol{v}\boldsymbol{X}_{j_0} = 1 \\
& \boldsymbol{u}\boldsymbol{Y}_j - \boldsymbol{w}\boldsymbol{Z}_j \le 0, j = 1,...,n \\
& \boldsymbol{w}\boldsymbol{Z}_j - \boldsymbol{v}\boldsymbol{X}_j \le 0, j = 1,...,n \\
& \boldsymbol{v} \ge 0, \boldsymbol{w} \ge 0, \boldsymbol{u} \ge 0
\end{aligned} \tag{3}
$$

Once an optimal solution $(\boldsymbol{v}^*, \boldsymbol{w}^*, \boldsymbol{u}^*)$ of model (3) is obtained, the overall efficiency and the stage efficiencies are calculated as follows:

$$
e_{j_o}^{o} = \boldsymbol{u}^*\boldsymbol{Y}_{j_0}, \quad e_{j_o}^{1} = \boldsymbol{w}^*\boldsymbol{Z}_{j_0}, \quad e_{j_o}^{2} = e_{j_o}^{o} \big/ e_{j_o}^{1} \tag{4}
$$

A major shortcoming of the multiplicative method is that the decomposition of the overall efficiency to the stage efficiencies is not unique. The authors have spotted many numerical examples that verify the non-uniqueness of the multiplicative decomposition.

2.2. The additive method

In the additive decomposition method[6], the overall efficiency and the stage efficiencies of the DMU_j are defined as follows:

$$
e_j^o = \frac{\boldsymbol{u}\boldsymbol{Y}_j + \boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j + \boldsymbol{w}\boldsymbol{Z}_j}, e_j^1 = \frac{\boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j}, e_j^2 = \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{w}\boldsymbol{Z}_j} \tag{5}
$$

The additive method differentiates in the definition of the overall efficiency. Notably, in (5) the intermediate measures appear in both terms of the fraction that defines the overall efficiency, meaning that they are considered as inputs and as outputs simultaneously. The decomposition model used is as follows:

$$
\frac{\boldsymbol{u}\boldsymbol{Y}_j + \boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j + \boldsymbol{w}\boldsymbol{Z}_j} = t_j^1 \frac{\boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j} + t_j^2 \frac{\boldsymbol{u}\boldsymbol{Y}_j}{\boldsymbol{w}\boldsymbol{Z}_j}; t_j^1 + t_j^2 = 1 \tag{6}
$$

i.e. the overall efficiency is the *weighted arithmetic average* of the stage efficiencies. The functional forms of the weights derive by solving the system (6) for t_j^1 and t_j^2, as follows:

$$
t_j^1 = \frac{\boldsymbol{v}\boldsymbol{X}_j}{\boldsymbol{v}\boldsymbol{X}_j + \boldsymbol{w}\boldsymbol{Z}_j}, \quad t_j^2 = \frac{\boldsymbol{w}\boldsymbol{Z}_j}{\boldsymbol{v}\boldsymbol{X}_j + \boldsymbol{w}\boldsymbol{Z}_j} \tag{7}
$$

Notably, as the weights are functions of the virtual measures, they depend on the unit being evaluated and, obviously, they generally differentiate from one unit to another. On the basis of the above definitions, the linear model below has been proposed to assess the overall efficiency of the evaluated unit j_0:

$$
\begin{aligned}
& e_{j_0}^{o} = \max \boldsymbol{u}\boldsymbol{Y}_{j_0} + \boldsymbol{w}\boldsymbol{Z}_{j_0} \\
& s.t. \\
& \boldsymbol{v}\boldsymbol{X}_{j_0} + \boldsymbol{w}\boldsymbol{Z}_{j_0} = 1 \\
& \boldsymbol{u}\boldsymbol{Y}_j - \boldsymbol{w}\boldsymbol{Z}_j \le 0, j = 1,...,n \\
& \boldsymbol{w}\boldsymbol{Z}_j - \boldsymbol{v}\boldsymbol{X}_j \le 0, j = 1,...,n \\
& \boldsymbol{v} \ge 0, \boldsymbol{w} \ge 0, \boldsymbol{u} \ge 0
\end{aligned} \tag{8}
$$

Once an optimal solution $(\boldsymbol{v}^*, \boldsymbol{w}^*, \boldsymbol{u}^*)$ of model (8) is obtained, the overall efficiency and the stage efficiencies are calculated as follows:

$$
\begin{aligned}
& e_{j_o}^{o} = \boldsymbol{u}^*\boldsymbol{Y}_{j_0} + \boldsymbol{w}^*\boldsymbol{Z}_{j_0} \\
& t_{j_0}^{1} = \boldsymbol{v}^*\boldsymbol{X}_{j_0}, \quad t_{j_0}^{2} = \boldsymbol{w}^*\boldsymbol{Z}_{j_0} \\
& e_{j_o}^{1} = \frac{\boldsymbol{w}^*\boldsymbol{Z}_{j_0}}{\boldsymbol{v}^*\boldsymbol{X}_{j_0}} = \frac{t_{j_0}^{2}}{t_{j_0}^{1}}, \quad e_{j_o}^{2} = \frac{e_{j_o}^{o} - t_{j_0}^{1} e_{j_o}^{1}}{t_{j_0}^{2}} = \frac{\boldsymbol{u}^*\boldsymbol{Y}_{j_0}}{\boldsymbol{w}^*\boldsymbol{Z}_{j_0}}
\end{aligned} \tag{9}
$$

In Chen et al.[6], the definition of the overall efficiency, as in (5), is implicit. Explicit is, however, the definition of the weights (7), which is made for the sake of linearization of the efficiency assessment model, to get model (8). However, as long as the weights derive from the optimal solution of (8), they depend on the DMU being evaluated and, generally, they are different for different DMUs. But this is not the only peculiarity emerging from the definition of the weights. Indeed, from the definition of the weights (7), as well as form (9) holds that $t_j^2 \le t_j^1$. This is a shortcoming of the additive decomposition model (8), as it biases the efficiency assessments in favor of the second stage.

3. The proposed approach

In this section we introduce a *bias-free* method to assess *unique* efficiency scores for the two stages, which are then are aggregated to get the overall efficiency score of the evaluated unit. Unlike the decomposition methods presented in the previous section, our method does not require an a priori definition of the overall efficiency. This grants our approach the flexibility to select the aggregation method a posteriori. Let us call this approach "*the composition approach*" as opposed to the decomposition approach.

3.1. Model development

Consider the output-oriented standard CRS-DEA model (10) for the first-stage and the input-oriented standard CRS-DEA model (11) for the second-stage, where the same intermediate weights are assumed for both stages:

$$\begin{aligned} & E_{j_0}^1 = \min \boldsymbol{vX}_{j_0} \\ & s.t. \\ & \boldsymbol{wZ}_{j_0} = 1 \\ & \boldsymbol{wZ}_j - \boldsymbol{vX}_j \leq 0, j = 1,...,n \\ & \boldsymbol{v} \geq 0, \boldsymbol{w} \geq 0 \end{aligned} \tag{10}$$

$$\begin{aligned} & E_{j_0}^2 = \max \boldsymbol{uY}_{j_0} \\ & s.t. \\ & \boldsymbol{wZ}_{j_0} = 1 \\ & \boldsymbol{uY}_j - \boldsymbol{wZ}_j \leq 0, j = 1,...,n \\ & \boldsymbol{w} \geq 0, \boldsymbol{u} \geq 0 \end{aligned} \tag{11}$$

As unifying the constraints of the two models above does not affect their optimal solutions, they can be jointly considered as a bi-objective linear program. Aggregating the two objective functions, we get the following single-objective linear program:

$$\begin{aligned} & \min \boldsymbol{vX}_{j_0} - \boldsymbol{uY}_{j_0} \\ & s.t. \\ & \boldsymbol{wZ}_{j_0} = 1 \\ & \boldsymbol{uY}_j - \boldsymbol{wZ}_j \leq 0, j = 1,...,n \\ & \boldsymbol{wZ}_j - \boldsymbol{vX}_j \leq 0, j = 1,...,n \\ & \boldsymbol{v} \geq 0, \boldsymbol{w} \geq 0, \boldsymbol{u} \geq 0 \end{aligned} \tag{12}$$

Once an optimal solution $(\boldsymbol{v}^*, \boldsymbol{w}^*, \boldsymbol{u}^*)$ of model (12) is obtained, the efficiency scores for unit j_0 in the first and the second stage are respectively:

$$\hat{e}_{j_0}^1 = \frac{1}{\boldsymbol{v}^*\boldsymbol{X}_{j_0}}, \hat{e}_{j_0}^2 = \boldsymbol{u}^*\boldsymbol{Y}_{j_0} \tag{13}$$

The unit j_0 is overall efficient, if and only if the optimal value of the objective function in (12) is zero. That is, our model (14) discriminates among overall efficient and inefficient units as, with respect to the external inputs $\boldsymbol{X}$ and the final outputs $\boldsymbol{Y}$, it is of the non-oriented additive DEA form. Model (14) assesses the efficiencies of the two stages without the need to assume weights for the two stages. Hence, our approach is "neutral", as opposed to the Chen's et al.[6] one, where the endogenous weights assumed for the individual stages favor the second stage against the first one. The individual efficiency scores obtained are unique, as model (14) locates a unique point on the Pareto-optimal frontier (in the multiobjective linear programming sense) in the value space ($\boldsymbol{vX}, \boldsymbol{wZ}, \boldsymbol{uY}$), at a minimum L_1 norm from the ideal point (1,1,1).

3.2. Aggregation of the individual efficiencies

As noticed in Liang et al.[9], it is reasonable to define the overall efficiency of the two-stage process as the simple arithmetic average of the efficiencies of the two individual stages. In this line of thought, the overall efficiency of unit j_0 is defined as:

$$\hat{e}^o_{j_0} = \tfrac{1}{2}\hat{e}^1_{j_0} + \tfrac{1}{2}\hat{e}^2_{j_0} \tag{14}$$

As the stage efficiencies are assumption-free, i.e. their assessment does not depend on any a priori definition of the overall efficiency, alternatively, they can be aggregated multiplicatively to get the overall efficiency as follows:

$$\hat{e}^o_{j_0} = \hat{e}^1_{j_0} \cdot \hat{e}^2_{j_0} = \frac{1}{\boldsymbol{v}^*\boldsymbol{X}_{j_0}} \cdot \boldsymbol{u}^*\boldsymbol{Y}_{j_0} = \frac{\boldsymbol{u}^*\boldsymbol{Y}_{j_0}}{\boldsymbol{v}^*\boldsymbol{X}_{j_0}} \tag{15}$$

3.3. Illustration

We apply our approach to the 24 Taiwanese non-life insurance companies originally studied in Kao and Hwang[5]. The authors consider a two-stage production process with two inputs (Operation expenses-X1 and Insurance expenses-X2), two intermediate measures (Direct written premiums-Z1 and Reinsurance premiums-Z2) and two final outputs (Underwriting profit-Y1 and Investment profit-Y2). Table 1 exhibits the data set.

Table 1. Taiwanese non-life insurance companies data set (source: Kao and Hwang[5])

#	Companies	X1	X2	Z1	Z2	Y1	Y2
1	Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	Shing kong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Allianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19	North America	159,422	182,338	1,141,950	483,291	519,121	46,857
20	Federal	145,442	53,518	316,829	131,920	355,624	26,537
21	Royal & Sunalliance	84,171	26,224	225,888	40,542	51,950	6,491
22	Aisa	15,993	10,502	52,063	14,574	82,141	4,181
23	AXA	54,693	28,408	245,910	49,864	0.1	18,980
24	Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

Table 2 (columns 2-5) displays the efficiency scores obtained by applying our model (12) on the data of Table 1 and the corresponding results reported in Chen et al.[6] along with the calculated weights (columns 6-10) and the results reported in Kao and Hwang[5] (columns 11-13). Although one can spot only a few differences in the individual efficiency scores in Table 2, in general, our approach does not yield the same efficiency scores for the individual stages with the other two methods. For instance, the stage-1 and stage-2 efficiency scores for DMU 16 (Allianz President) differ substantially from those obtained from the additive decomposition method. As regards the results obtained from the multiplicative decomposition method, the individual efficiency scores are different for 9 of the 24 units.

Table 2. Results from model (12) compared to Chen et al.[6] and Kao and Hwang[5]

	Our model (12)				Chen et al.[6]					Kao and Hwang[5]		
DMU	$\hat{e}^1$	$\hat{e}^2$	$\hat{e}^o=(\hat{e}^1+\hat{e}^2)/2$	$\hat{e}^o=\hat{e}^1\cdot\hat{e}^2$	e^1	e^2	e^o	t^1	t^2	e^1	e^2	e^o
1	0.993	0.704	0.849	0.699	0.993	0.704	0.849	0.502	0.498	0.993	0.704	0.699
2	0.998	0.626	0.812	0.625	0.998	0.626	0.812	0.500	0.500	0.998	0.626	0.625
3	0.690	1	0.845	0.690	0.690	1	0.817	0.592	0.408	0.690	1	0.690
4	0.724	0.420	0.572	0.304	0.724	0.420	0.596	0.580	0.420	0.724	0.420	0.304
5	0.831	0.923	0.877	0.767	0.831	0.923	0.873	0.546	0.454	0.831	0.923	0.767
6	0.961	0.406	0.683	0.390	0.961	0.406	0.689	0.510	0.490	0.961	0.406	0.390
7	0.752	0.352	0.552	0.265	0.752	0.352	0.580	0.571	0.429	0.671	0.412	0.277
8	0.726	0.378	0.552	0.274	0.726	0.378	0.579	0.580	0.420	0.663	0.415	0.275
9	1	0.223	0.612	0.223	1	0.223	0.612	0.500	0.500	1	0.223	0.223
10	0.862	0.541	0.701	0.466	0.862	0.541	0.713	0.537	0.463	0.862	0.541	0.466
11	0.729	0.207	0.468	0.151	0.729	0.207	0.509	0.578	0.422	0.647	0.253	0.164
12	1	0.760	0.880	0.760	1	0.760	0.880	0.500	0.500	1	0.760	0.760
13	0.811	0.243	0.527	0.197	0.811	0.243	0.557	0.552	0.448	0.672	0.309	0.208
14	0.725	0.374	0.549	0.271	0.725	0.374	0.577	0.580	0.420	0.670	0.431	0.289
15	1	0.614	0.807	0.614	1	0.614	0.807	0.500	0.500	1	0.614	0.614
16	0.907	0.336	0.621	0.304	0.886	0.362	0.639	0.530	0.470	0.886	0.362	0.320
17	0.723	0.460	0.591	0.332	0.723	0.460	0.613	0.580	0.420	0.628	0.574	0.360
18	0.794	0.326	0.560	0.259	0.794	0.326	0.587	0.558	0.442	0.794	0.326	0.259
19	1	0.411	0.706	0.411	1	0.411	0.706	0.500	0.500	1	0.411	0.411
20	0.933	0.586	0.759	0.547	0.933	0.586	0.765	0.517	0.483	0.933	0.586	0.547
21	0.751	0.262	0.506	0.197	0.751	0.262	0.541	0.571	0.429	0.732	0.274	0.201
22	0.590	1	0.795	0.590	0.590	1	0.742	0.629	0.371	0.590	1	0.590
23	0.843	0.499	0.671	0.420	0.843	0.499	0.685	0.543	0.457	0.843	0.499	0.420
24	1	0.087	0.544	0.087	1	0.087	0.544	0.500	0.500	0.429	0.314	0.135

Our experiments with different randomly generated data sets (100 data sets drawn from a uniform distribution, with 50 DMUs, 2 external inputs, 3 intermediate measures and 2 final outputs) revealed significant differentiation in the efficiency results between the three methods. The percentage of units in each run that showed different stage efficiency scores, with respect to model (12) and the additive model (8) varied from 0% to 82%. In only one case the efficiency scores were identical for all the units. Analogously, the percentage of units in each run that showed different individual efficiency scores, with respect to model (12) and the multiplicative model (3) varied from 23% to 97%. None case was spotted with identical efficiency scores for all the units.

One can observe in Table 2 that $\hat{e}^1 \geq e^1$ and $\hat{e}^2 \leq e^2$ where e^1 and e^2 are the stage-1 and stage-2 efficiency scores derived by either the additive or the multiplicative models (8) and (3) respectively. These relations are completely verified throughout our experiments. As concerns the additive decomposition model (8), this is empirical evidence that the efficiency assessments are biased in favor of the second stage.

4. Conclusion

We introduced in this paper a novel approach to assess the individual and the overall efficiencies in two-stage DEA, under the common assumption of series relationship between the two stages that permeates the literature. Our approach effectively overcomes the shortcomings highlighted for the additive decomposition method (biased efficiency assessments) and the multiplicative decomposition method (non-unique efficiency assessments), by providing unique and unbiased efficiency scores for the two stages. Based on a reverse perspective in aggregating the individual efficiency scores, i.e. the composition as opposed to the decomposition approach, we estimate first the individual efficiencies for the two stages, which then can be aggregated in either an additive or a multiplicative form to obtain the overall efficiency. Our modeling approach is based on the selection of an output orientation for the first stage and an input orientation for the second stage. In this manner, the intermediate measures are used as the basis to link the efficiency assessment models for the two stages in a single linear program.

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