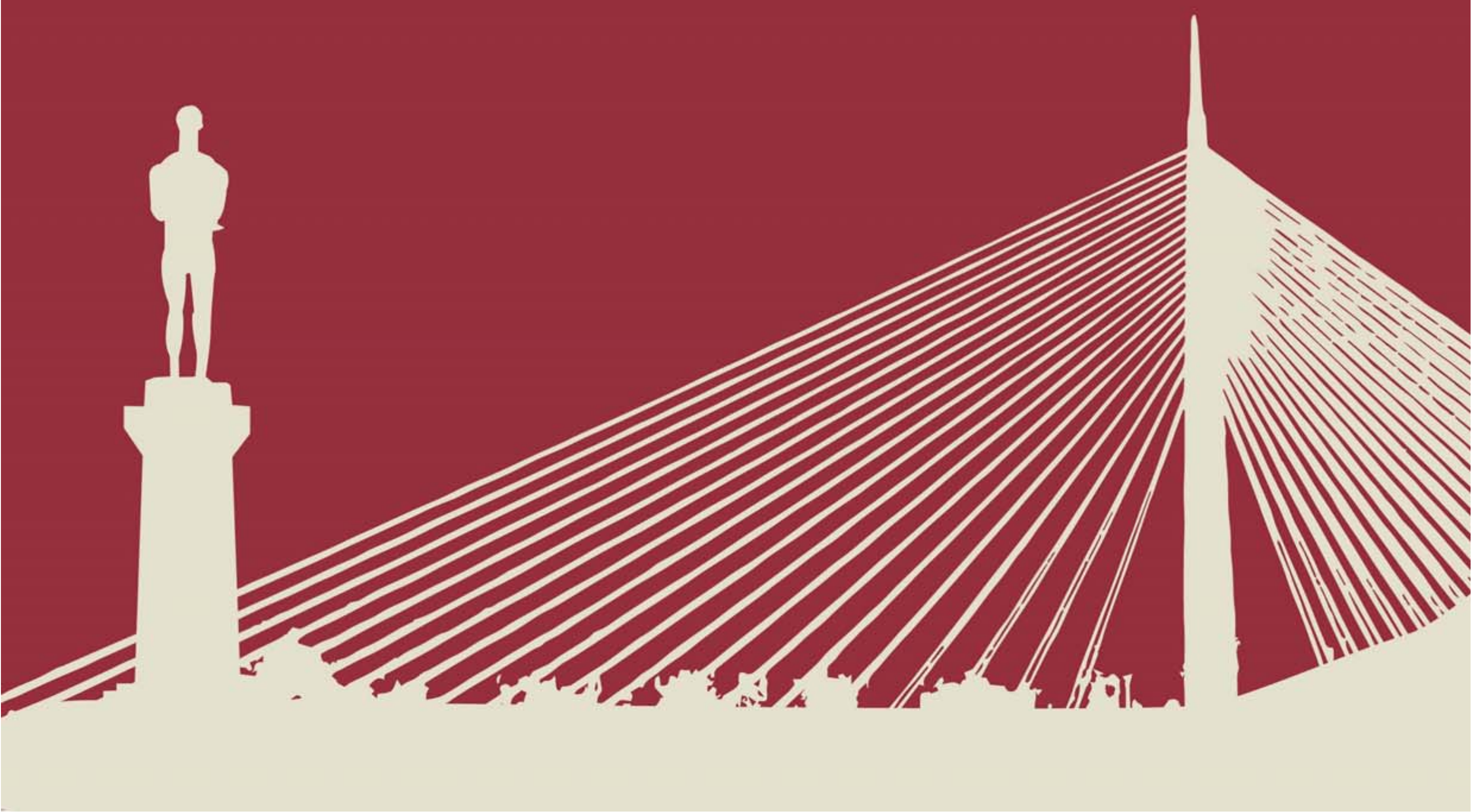
BALCOR 2013
Belgrade – Zlatibor, Serbia
7-11 September, 2013

XI Balkan Conference on Operational Research

Conference Proceedings



XI Balkan Conference on Operational Research
BALCOR 2013
Belgrade, Zlatibor, 07-10 September 2013
UNIVERSITY OF BELGRADE FACULTY OF ORGANISATIONAL SCIENCES

INCORPORATING USER PREFERENCES IN DEA WITH ORDINAL REGRESSION

DIMITRIS SOTIROS[1], DIMITRIS K. DESPOTIS[2], GREGORY KORONAKOS[3], YIANNIS G. SMIRLIS[4]

[1] Department of Informatics, University of Piraeus, Greece, dsotiros@unipi.gr
[2] Department of Informatics, University of Piraeus, Greece, despotis@unipi.gr
[3] Department of Informatics, University of Piraeus, Greece, gkoron@unipi.gr
[4] Department of Informatics, University of Piraeus, Greece, smirlis@unipi.gr

Abstract: *Value-based DEA is a recent development that resorts to value assessment protocols from multiple criteria decision analysis (MCDA) to transform the original input/output data to a value scale, thus driving the efficiency assessments in line with individual preferences. In this paper, we present a piece-wise linear programming approach to value-based DEA by employing a data transformation-variable alteration technique and assurance region constraints. We propose a hybrid approach to incorporate the decision maker's value functions in the context of DEA through ordinal regression.*

Keywords: *Data Envelopment Analysis (DEA), Value-based DEA, Ordinal Regression*

1. INTRODUCTION

Data Envelopment Analysis (DEA) is a widespread technique for measuring the relative efficiency of decision making units (DMUs) on the basis of multiple inputs and outputs (Charnes *et al.* 1978, Banker *et al.* 1984). One of the major advantages of DEA is that each unit is free to select the weights assigned to the factors so as to maximize its relative efficiency score. However, as the weights may not be in line with the individual preferences of a decision maker, several approaches have been arisen to incorporate value judgments in the context of DEA.

In the DEA literature, there are two classes of methods to incorporate value judgments. The first one refers to the restriction of the weight space (Dyson and Thanassoulis 1988, Cook *et al.* 1991) and the second one to the alteration of the data space either by altering the data set itself (Charnes *et al.* 1989) or by introducing fictitious DMUs (Golany and Roll 1994, Thanassoulis and Allen 1998, Podinovski 2004). Gouveia *et al.* (2008) and Almeida and Dias (2012) used a value assessment protocol from multiple criteria decision analysis (MCDA) to transform the original input/output data to a value scale and then, they performed the efficiency assessments with an weighted additive DEA model. Recently, Despotis and Sotiros (2013) introduced a piece-wise linear programming approach to value-based DEA. Applying a data transformation-variable alteration technique they developed an oriented value-based DEA model with assurance region constraints capable of assessing a radial measure of efficiency. They used their model as an alternative approach to the case studied in Almeida and Dias (2012) on the basis of the same value assessment protocol.

In this paper, building on Despotis and Sotiros (2013) work, we propose a two-phase hybrid approach to incorporate user preferences in DEA efficiency assessments. In particular, we present a data transformation - variable alteration technique that enable us to employ ordinal regression analysis by the UTASTAR method (Jacquet-Lagreze and Siskos 1982, Siskos and Yannacopoulos 1985), so as to obtain value functions according to the user's preferences. These value functions are then incorporated in our value-based DEA model by imposing restrictions on the value variables.

The rest of the paper unfolds as follows. In section 2, we reformulate the piece-wise linear DEA model introduced by Despotis and Sotiros (2013) to enable the exploitation of the preferential model assessed by the ordinal regression method UTASTAR. In section 3, we present the value assessment procedure based on UTASTAR. Section 4 outlines the proposed two-phase approach. Conclusions are given in section 5.

2. A VALUE-BASED DEA MODEL

Consider n DMUs that use m inputs to produce s outputs and let $Y_j = (y_{1j}, y_{2j}, ..., y_{sj})$ and $X_j = (x_{1j}, x_{2j}, ..., x_{mj})$ be the output and input vectors of unit j respectively. Then, the overall value

$U\left(Y_j\right)$ of the outputs of unit j is given by the additive value function $U(Y_j)=\sum_{r=1}^{s}U_r(y_{rj})$, where U_r, $r=1,...,s$ represent non-decreasing partial value functions. Similarly, the overall value $V\left(X_j\right)$ of the inputs of unit j is given by $V(X_j)=\sum_{i=1}^{m}V_i(x_{ij})$, where V_i, $i=1,...,m$ represent non-increasing value functions. These value functions are characterized as generalizations of the total virtual outputs and inputs, which are assumed linear in standard DEA:

$$U(Y_j)=\sum_{r=1}^{s}y_{rj}u_r\ ,\ V(X_j)=\sum_{i=1}^{m}x_{ij}v_i \tag{1}$$

where u_r, $r=1,2,...,s$ and v_i, $i=1,2,...,m$ are the weighting variables assigned to the outputs and the inputs respectively.

Assuming non-increasing value functions for the inputs, allows us to treat the inputs as outputs. Then, the value-based relative efficiency E_{j_0} of the evaluated unit j_0 is estimated by the following general model, which is equivalent to an input-oriented DEA model with $m+s$ outputs and a dummy input, set at the level of 1 for all the units.

$$\begin{aligned}&\max E_{j_0}=\sum_{r=1}^{s}U_r(y_{rj_0})+\sum_{i=1}^{m}V_i(x_{ij_0})\\&s.t.\\&\sum_{r=1}^{s}U_r(y_{rj})+\sum_{i=1}^{m}V_i(x_{ij})\le 1\quad (j=1,...,n)\end{aligned} \tag{2}$$

Differentiating from the standard DEA, we assume non-linear value functions in the DEA efficiency assessments. Relaxing the linearity assumption, allows us to treat cases where, for example, the marginal value of an output diminishes as the output increases. We build to this end on Cook and Zhu (2009) and Despotis *et a*l. (2010), who introduced piece-wise linear representations for non-linear virtual inputs and outputs in DEA.

2.1 Modeling piece-wise linear value functions for outputs

Let $l_r \le \min_j\{y_{rj}\}$ and $h_r \ge \max_j\{y_{rj}\}$ be fixed minimum and maximum values for output *r*, set so as the range $[l_r,h_r]\supseteq[\min_j\{y_{rj}\},\max_j\{y_{rj}\}]$ covers the observed outputs of the entire set of units. We assume k_r+1 breakpoints that split the range [l_r, h_r] of the output *r* in k_r segments: $[b_r^1,b_r^2],[b_r^2,b_r^3],...,[b_r^{k_r},b_r^{k_r+1}]$, with $b_r^1=l_r$ and $b_r^{k_r+1}=h_r$. By convention, we set $U_r(l_r)=0$. Under this segmentation, any output $y_{rj}\in[l_r,h_r]$ can be decomposed as $y_{rj}=l_r+\delta_{rj}^1+\delta_{rj}^2+...+\delta_{rj}^{k_r}$, where

$$\delta_{rj}^1=\begin{cases}y_{rj}\text{-}b_r^1 & if\ \ y_{rj}\le b_r^2\\ b_r^2\text{-}b_r^1 & if\ \ y_{rj}>b_r^2\end{cases}$$
$$\delta_{rj}^{\mu}=\begin{cases}0 & if\ \ y_{rj}\le b_r^{\mu}\\ y_{rj}-b_r^{\mu} & if\ \ b_r^{\mu}<y_{rj}\le b_r^{\mu+1}\\ b_r^{\mu+1}-b_r^{\mu} & if\ \ y_{rj}>b_r^{\mu+1}\end{cases}\quad ,\ \mu=2,3,...,k_r-1 \tag{3}$$

$$\delta_{rj}^{k_r} = \begin{cases} 0 & if \ \ y_{rj} \le b_r^{k_r} \\ y_{rj} - b_r^{k_r} & if \ \ b_r^{k_r} < y_{rj} \le b_r^{k_r+1} \end{cases}$$

Assuming that the value function is linear in each segment, we assign a distinct weight variable $u_{r\mu}$ to each segment $\mu = 1, 2, ..., k_r$. Then, the partial value $U_r(y_{rj})$ is given in a piece-wise linear form as:

$$U_r(y_{rj}) = \delta_{rj}^1 u_{r1} + \delta_{rj}^2 u_{r2} + ... + \delta_{rj}^{k_r} u_{rk_r} = \sum_{\mu=1}^{k_r} \delta_{rj}^{\mu} u_{r\mu} \tag{4}$$

Applying to each segment the transformation

$$\delta_{rj}^{\mu} u_{r\mu} = \frac{\delta_{rj}^{\mu}}{\left(b_r^{\mu+1} - b_r^{\mu}\right)} \left(b_r^{\mu+1} - b_r^{\mu}\right) u_{r\mu} = \hat{\delta}_{rj}^{\mu} p_{r\mu}, \ \mu = 1, 2, ..., k_r$$

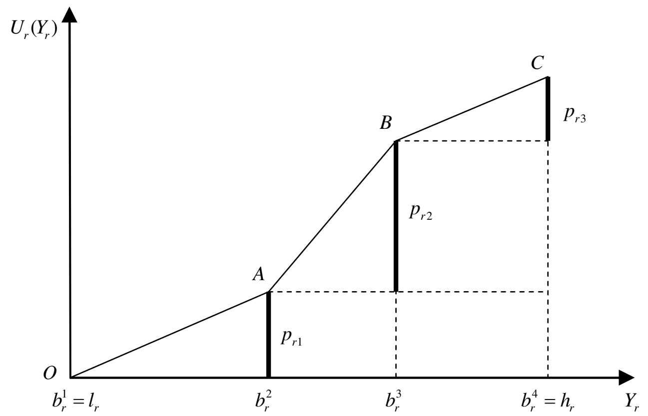
the partial value function (4) is expressed in terms of the new variables $p_{r1}, p_{r2}, ..., p_{rk_r}$ as follows:

$$U_r(y_{rj}) = \hat{\delta}_{rj}^1 p_{r1} + \hat{\delta}_{rj}^2 p_{r2} + ... + \hat{\delta}_{rj}^{k_r} p_{rk_r} = \sum_{\mu=1}^{k_r} \hat{\delta}_{rj}^{\mu} p_{r\mu} \tag{5}$$

with $\hat{\delta}_{rj}^{\mu} = \dfrac{\delta_{rj}^{\mu}}{b_r^{\mu+1} - b_r^{\mu}}$, $\mu = 1, 2, ..., k_r$

It is straightforward from (5) that $U_r(h_r) = p_{r1} + p_{r1} + ... + p_{rk_r}$ where $p_{r\mu}$, $\mu = 1, ..., k_r$ represent the increment of worth in the interval $[b_r^{\mu}, b_r^{\mu+1}]$ (Figure 1).

Figure 1: Value function for a non-linear output measure Y_r.



After the above transformations, the value function (total virtual output) of unit *j* can be expressed as follows:

$$U(Y_j) = \sum_{r=1}^{s} U_r(y_{rj}) = \sum_{r=1}^{s} \sum_{\mu=1}^{k_r} \hat{\delta}_{rj}^{\mu} p_{r\mu} \tag{6}$$

2.2 Modeling piece-wise linear value functions for inputs

Analogously, we model non-increasing non-linear value functions for the inputs in a piece-wise linear form. Let $l_i \le \min_j\{x_{ij}\}$ and $h_i \ge \max_j\{x_{ij}\}$ be fixed minimum and maximum values for input *i*, set so as the range $[l_i, h_i] \supseteq [\min_j\{x_{ij}\}, \max_j\{x_{ij}\}]$ covers the observed inputs of the entire set of units. By convention, we set

$V_i(h_i)=0$. Taking a number of k_i+1 breakpoints that split the interval $[l_i,\ h_i]$ in k_i segments $[a_i^1,a_i^2],[a_i^2,a_i^3],...,[a_i^{k_i},a_i^{k_i+1}]$, with $a_i^1=l_i$ and $a_i^{k_i+1}=h_i$, we decompose any input value $x_{ij}\in[l_i,h_i]$ to $x_{ij}=h_i-(\gamma_{ij}^1+\gamma_{ij}^2+...+\gamma_{ij}^{k_i})$ where:

$$\gamma_{ij}^1=\begin{cases}0 & if\ x_{ij}\geq a_i^2\\ a_i^2-x_{ij} & if\ a_i^1\leq x_{ij}<a_i^2\end{cases}$$

$$\gamma_{ij}^{\mu}=\begin{cases}0 & if\ x_{ij}\geq a_i^{\mu+1}\\ a_i^{\mu+1}-x_{ij} & if\ a_i^{\mu}\leq x_{ij}<a_i^{\mu+1}\\ a_i^{\mu+1}-a_i^{\mu} & if\ x_{ij}<a_i^{\mu}\end{cases},\ \mu=2,3,...,k_i-1 \tag{7}$$

$$\gamma_{ij}^{k_i}=\begin{cases}a_i^{k_i+1}-x_{ij} & if\ x_{ij}\geq a_i^{k_i}\\ a_i^{k_i+1}-a_i^{k_i} & if\ x_{ij}<a_i^{k_i}\end{cases}$$

Then, the partial value $V_i(x_{ij})$ can be expressed in the following piece-wise linear form:

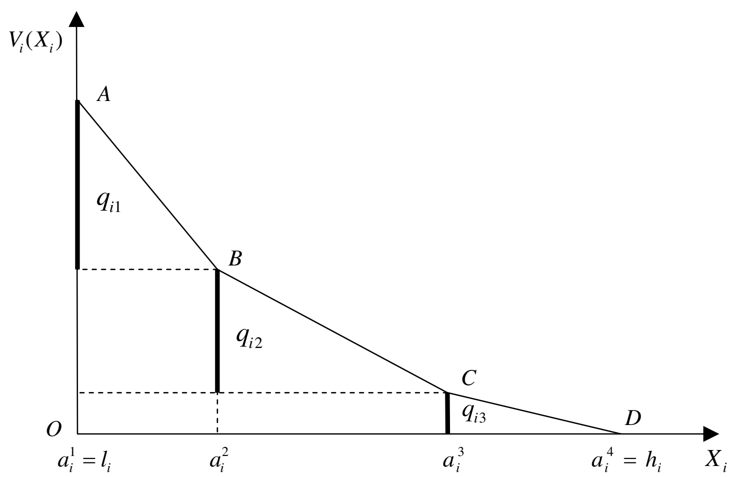
$$V_i(x_{ij})=\gamma_{ij}^1 v_{i1}+\gamma_{ij}^2 v_{i2}+...+\gamma_{ij}^{k_i} v_{ik_i}=\sum_{\mu=1}^{k_i}\gamma_{ij}^{\mu} v_{i\mu} \tag{8}$$

where $v_{i\mu}$ represent distinct weight variables assigned to each segment $\mu=1,2,...,k_i$. Applying to each segment the same transformation introduced for piece-wise linear outputs, we derive the value function (8) in terms of the new variables $q_{i1},q_{i2},...,q_{ik_i}$ as follows:

$$V_i(x_{ij})=\hat{\gamma}_{ij}^1 q_{i1}+\hat{\gamma}_{ij}^2 q_{i2}+...+\hat{\gamma}_{ij}^{k_i} q_{ik_i}=\sum_{\mu=1}^{k_i}\hat{\gamma}_{ij}^{\mu} q_{i\mu} \tag{9}$$

with $\hat{\gamma}_{ij}^{\mu}=\dfrac{\gamma_{ij}^{\mu}}{a_i^{\mu+1}-a_i^{\mu}}$, $\mu=1,2,...,k_i$. It is straightforward from (9) that $V_i(l_i)=q_{i1}+q_{i2}+...+q_{ik_i}$ where $q_{i\mu}$ represent the decrement of worth in the interval $[a_i^{\mu},a_i^{\mu+1}]$, $\mu=1,...,k_i$ (Figure 2).

Figure 2: Value function for a non-linear input measure X_i.



Exploiting the above transformations, the value function (total virtual input) of unit j can be expressed as follows:

$$V(X_j)=\sum_{i=1}^{m} V_i(x_{ij})=\sum_{i=1}^{m}\sum_{\mu=1}^{k_i}\hat{\gamma}_{ij}^{\mu} q_{i\mu} \tag{10}$$

According to the equations (6) and (10) and the general model (2), the efficiency score E_{j_0} of unit j_0 can be assessed by the following linear model:

$$\begin{aligned}
&\max E_{j_0}(p,q)=\sum_{r=1}^{s}\sum_{\mu=1}^{k_r}\hat{\delta}_{rj_o}^{\mu} p_{r\mu}+\sum_{i=1}^{m}\sum_{\mu=1}^{k_i}\hat{\gamma}_{ijo}^{\mu} q_{i\mu}\\
&s.t.\\
&\sum_{r=1}^{s}\sum_{\mu=1}^{k_r}\hat{\delta}_{rj}^{\mu} p_{r\mu}+\sum_{i=1}^{m}\sum_{\mu=1}^{k_i}\hat{\gamma}_{ij}^{\mu} q_{i\mu}\le 1 \;\; j=1,2,...,n\\
&p_{r\mu}\ge 0 \;\; (r=1,...,s,\;\; \mu=1,...,k_r)\\
&q_{i\mu}\ge 0 \;\; (i=1,...,m,\;\; \mu=1,...,k_i)\\
&p_{r\mu},q_{i\mu}\in W
\end{aligned} \tag{11}$$

In the last constraint of (11), W denotes the region defined by additional restrictions on the variables, which derive by translating the value functions assessed with respect to the user's preferences. The assessment is made by the ordinal regression method UTASTAR and is described in the next section.

3. ASSESSING VALUE FUNCTIONS FOR DEA WITH THE UTASTAR METHOD

The ordinal regression method UTASTAR (Siskos and Yannacopoulos, 1985) is an extension of UTA multi criteria method (Jacquet-Lagreze and Siskos, 1982), which is based on linear programming. It adopts the aggregation-disaggregation principle in order to assess value functions according to the decision makers' preferential structure. Given a weak preference order on a subset of alternatives that the decision maker is familiar with, the value functions of the criteria are adjusted so as to develop a preference model as consistent as possible with the decision maker's preferences.

In this section, we develop a two-phase approach that bridges UTASTAR with DEA. Adjusting the UTASTAR formulation so as to be compatible with the developments presented in the previous section, we apply, in phase I, the UTASTAR method to assess the preferential model of the user. Then, the assessed model is incorporated, in phase II, in the DEA efficiency assessments. A regular interpretation of DEA inputs and outputs to criteria in the MCDA terminology is that inputs are criteria to be minimized, whereas outputs are criteria to be maximized. With such a correspondence, the formulations in (6) and (10) developed in the previous section can be fully utilized in the UTASTAR context.

Given a subset A_R of the n DMUs and a weak order on its items, that reflects the user's overall preference over A_R , the LP model below assess piece-wise linear functions for the criteria (inputs and outputs) as consistent as possible with the user's stated preferences:

$$\begin{aligned}
&\min\ \mathrm{F}=\sum_{j\in A_R}\left(d_j^{+}+d_j^{-}\right)\\
&s.t.\\
&\left(E_j(p,q)+d_j^{-}-d_j^{+}\right)-\left(E_{j+1}(p,q)+d_{j+}^{-}-d_{j+1}^{+}\right)>\delta \;\; if\;\, jPj+1\\
&\left(E_j(p,q)+d_j^{-}-d_j^{+}\right)-\left(E_{j+1}(p,q)+d_{j+}^{-}-d_{j+1}^{+}\right)=0 \;\; if\;\, jIj+1\\
&\sum_{r=1}^{s}\sum_{\mu=1}^{k_r} p_{r\mu}+\sum_{i=1}^{m}\sum_{\mu=1}^{k_i} q_{i\mu}=1\\
&p_{r\mu}\ge 0, q_{i\mu}\ge 0, d_j^{-}\ge 0, d_j^{+}\ge 0, j\in A_R
\end{aligned} \tag{12}$$

where d_j^{+}, d_j^{-} are overestimation and underestimation errors respectively, P denotes strict preference and I denotes indifference. As model (12) may have multiple optimal solutions, characteristic optimal solutions are investigated that maximize the value of one criterion at a time. This is accomplished in a post-optimality stage by solving the following LP for each criterion (input and output):

$$\max\ \varphi_r = \sum_{\mu=1}^{k_r} p_{r\mu} \quad (r=1,...,s)$$
$$s.t.$$
$$\left(E_j(p,q)+d_j^- - d_j^+\right) - \left(E_{j+1}(p,q)+d_{j+}^- - d_{j+1}^+\right) > \delta \ if\ jPj+1$$
$$\left(E_j(p,q)+d_j^- - d_j^+\right) - \left(E_{j+1}(p,q)+d_{j+1}^- - d_{j+1}^+\right) = 0 \ if\ jIj+1 \tag{13}$$
$$\sum_{r=1}^{s}\sum_{\mu=1}^{k_r} p_{r\mu} + \sum_{i=1}^{m}\sum_{\mu=1}^{k_i} q_{i\mu} = 1$$
$$\sum_{j\in A_R}\left(d_j^+ + d_j^-\right) \le F^*$$
$$p_{r\mu}\ge 0, q_{i\mu}\ge 0, d_j^-\ge 0, d_j^+\ge 0, j\in A_R$$

Totally, s+m LPs are solved (i.e. s LPs for the criteria associated with the outputs and m LPs for the criteria associated with the inputs). Model (13) is presented for the outputs. The model can be adjusted for inputs by replacing the objective function with $\varphi_i = \sum_{\mu=1}^{k_i} q_{i\mu}$, i=1,…,m. The last constraint in (13) is introduced in order to support the optimal value F^* of the objective function attained in model (12). Having obtained s+m alternative optimal solutions, the average, which is also optimal due to convexity, is used as representative of the user's preferences. This completes the phase I of our approach.

If $\left(\tilde{p}_{11}, \tilde{p}_{12},..., \tilde{p}_{1k_1},...., \tilde{p}_{s1}, \tilde{p}_{s2},..., \tilde{p}_{sk_s}, \tilde{q}_{11}, \tilde{q}_{12},..., \tilde{q}_{1k_1},...., \tilde{q}_{m1}, \tilde{q}_{m2},..., \tilde{q}_{mk_m}\right)$ denote the average optimal solution, the assessed preferential model is incorporated in the DEA model (11), by appending the following constraint set:

$$W = \begin{cases} \dfrac{p_{r,\mu+1}}{p_{r,\mu}} = \dfrac{\tilde{p}_{r,\mu+1}}{\tilde{p}_{r,\mu}}, \ r=1,...,s; \mu=1,...,k_r \\ \dfrac{q_{i,\mu+1}}{q_{i,\mu}} = \dfrac{\tilde{q}_{i,\mu+1}}{\tilde{q}_{i,\mu}}, \ i=1,...,m; \mu=1,...,k_i \end{cases} \tag{14}$$

Solving model (11) with the additional constraints (14) for one DMU at a time we get the efficiency scores of the entire set of DMUs. This is the phase II, which completes approach.

4. ILLUSTRATION

We provide in this section a numerical illustration with 25 DMUs, one input (X_1) and two outputs (Y_1, Y_2) as depicted in Table 1.

Table 1: Observed input/output data in original scales

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
X_1	60	54	57	50	64	40	60	47	52	61	65	44	68	62	54	53	53	52	55	56	63	70	60	53	61
Y_1	57	45	64	58	56	60	58	46	53	54	50	72	46	50	48	43	53	52	70	53	44	71	67	53	40
Y_2	51	53	58	52	59	59	42	46	48	50	52	55	25	50	67	44	64	59	55	57	39	54	71	56	55

For the sake of simplicity, we assume three breakpoints for each factor as shown in Table 2 (i.e. the range of each factor is split in two segments)

Table 2: Breakpoints for the non-linear inputs and outputs in original scales

Factors	b_1	b_2	b_3	α_1	α_2	α_3
X_1	-	-	-	40	60	75
Y_1	30	50	72	-	-	-
Y_2	10	40	71	-	-	-

Applying the data transformation-variable alteration introduced in section 2, we obtain the expanded data set in original scales and its range-normalized counterpart, as shown in Table 3 and Table 4 respectively.

Table 3: Expanded data set in original scales

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
γ_1^1	0	6	3	10	0	20	0	13	8	0	0	16	0	0	6	7	7	8	5	4	0	0	0	7	0
γ_1^2	15	15	15	15	11	15	15	15	15	14	10	15	7	13	15	15	15	15	15	15	12	5	15	15	14
δ_1^1	20	15	20	20	20	20	20	16	20	20	20	20	16	20	18	13	20	20	20	20	14	20	20	20	10
δ_1^2	7	0	14	8	6	10	8	0	3	4	0	22	0	0	0	0	3	2	20	3	0	21	17	3	0
δ_2^1	30	30	30	30	30	30	30	30	30	30	30	30	15	30	30	30	30	30	30	30	29	30	30	30	30
δ_2^2	11	13	18	12	19	19	2	6	8	10	12	15	0	10	27	4	24	19	15	17	0	14	31	16	15

Table 4: Range-normalized data set

DMU	$\hat{\gamma}_1^1$	$\hat{\gamma}_1^2$	$\hat{\delta}_1^1$	$\hat{\delta}_1^2$	$\hat{\delta}_2^1$	$\hat{\delta}_2^2$
1	0	1	1	0.32	1	0.35
2	0.30	1	0.75	0	1	0.42
3	0.15	1	1	0.64	1	0.58
4	0.50	1	1	0.36	1	0.39
5	0	0.73	1	0.27	1	0.61
6	1	1	1	0.45	1	0.61
7	0	1	1	0.36	1	0.06
8	0.65	1	0.80	0	1	0.19
9	0.40	1	1	0.13	1	0.26
10	0	0.93	1	0.18	1	0.32
11	0	0.68	1	0	1	0.39
12	0.80	1	1	1	1	0.48
13	0	0.47	0.80	0	0.50	0
14	0	0.87	1	0	1	0.32
15	0.30	1	0.90	0	1	0.87
16	0.35	1	0.65	0	1	0.13
17	0.35	1	1	0.14	1	0.77
18	0.40	1	1	0.09	1	0.61
19	0.25	1	1	0.91	1	0.48
20	0.20	1	1	0.14	1	0.55
21	0	0.80	0.70	0	0.97	0
22	0	0.33	1	0.95	1	0.45
23	0	1	1	0.77	1	1
24	0.35	1	1	0.14	1	0.52
25	0	0.93	0.50	0	1	0.48

Table 5 below depicts the selected subset A_R with the preference ranking provided by a hypothetical user.

Table 5: A subset of DMUs and a preference ranking

DMU	$\hat{\gamma}_1^1$	$\hat{\gamma}_1^2$	$\hat{\delta}_1^1$	$\hat{\delta}_1^2$	$\hat{\delta}_2^1$	$\hat{\delta}_2^2$	Ranking
6	1	1	1	0.45	1	0.61	1
12	0.80	1	1	1	1	0.48	2
19	0.25	1	1	0.91	1	0.48	3
3	0.15	1	1	0.64	1	0.58	4
15	0.30	1	0.90	0	1	0.87	5
1	0	1	1	0.32	1	0.35	6
21	0	0.80	0.70	0	0.97	0	7

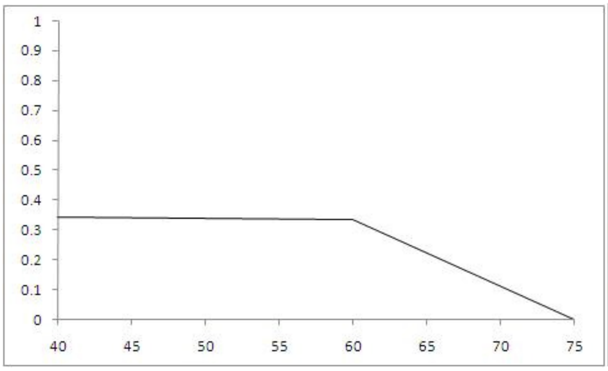
Applying model (12) and then performing the post-optimality analysis with models (13) on the data of Table 5, we get the following average optimal solution (Table 6) with $F^*=0$.

Table 6: Optimal solution acquired from ordinal regression analysis

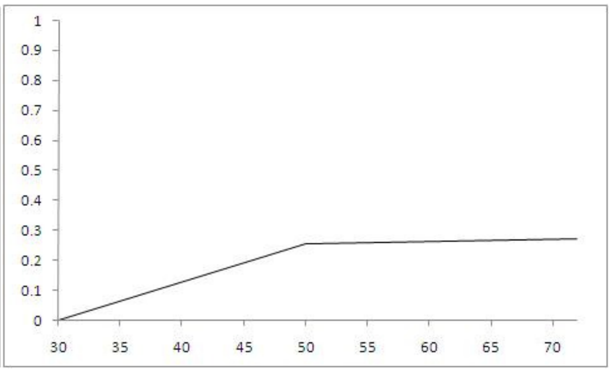
$\tilde{q}_{11}$	$\tilde{q}_{12}$	$\tilde{p}_{11}$	$\tilde{p}_{12}$	$\tilde{p}_{21}$	$\tilde{p}_{22}$
0.009	0.333	0.255	0.016	0.333	0.054

As the optimal value of the objective function in model (12) is zero ($F^*=0$), the assessed preference model is fully consistent with the ranking provided by the user. Figure 3 depicts the value functions assessed for the input X_1 and the outputs Y_1 and Y_2 on the basis of the optimal solution given in Table 6.

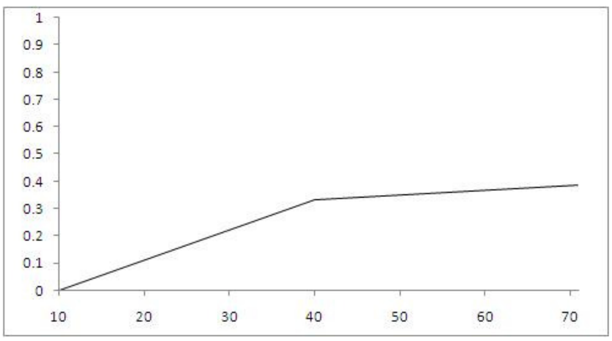
Figure 3: Value functions for the input X_1 and the outputs Y_1 and Y_2.



X_1



Y_1



Y_2

The assessment of the value functions completes the phase I of our approach. The value-based DEA efficiency assessments are made in phase II, by incorporating in the DEA model (11) the following set of constraints

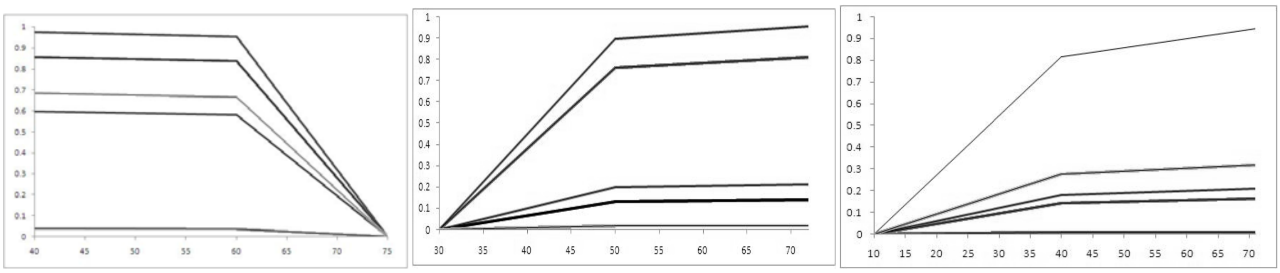
$$W = \begin{cases} 0.009q_{12} - 0.333q_{11} = 0 \\ 0.255p_{12} - 0.016p_{11} = 0 \\ 0.333p_{22} - 0.054p_{21} = 0 \end{cases}$$

which translate the assessed value functions in terms of the variables *p* and *q*. The efficiency scores of the units, as shown in Table 7, are obtained by solving model (11) for one DMU at a time. Figure 4 exhibits the contribution of the input and the outputs to the efficiency index, as assessed by each evaluated DMU in order to maximize its efficiency score. This is the major characteristic DEA, which grants the flexibility to each DMU to assess its efficiency score by putting higher value to its advantageous features (inputs or outputs). As shown in Figure 4, the underlying value functions assumed by all the DMUs in the DEA efficiency assessments maintain the preferential model assessed by UTASTAR on a sample of DMUs (Figure 3).

Table 7: Efficiency scores according to model (11) and the above restrictions

DMU	efficiency	DMU	efficiency
1	0.977	*14*	0.942
2	0.978	*15*	0.997
3	0.988	*16*	0.977
4	0.988	*17*	0.995
5	0.957	*18*	0.989
6	1.000	*19*	0.995
7	0.977	*20*	0.983
8	0.987	*21*	0.828
9	0.984	*22*	0.971
10	0.954	*23*	1.000
11	0.936	*24*	0.984
12	1.000	*25*	0.924
13	0.739		

Figure 4: Value functions assessed by the DMUs for X_1, Y_1 and Y_2 .



X_1 Y_1 Y_2

5. CONCLUSION

We present in this paper, a two-phase approach to incorporate user preferences in DEA efficiency assessments. Our modeling approach bridge the ordinal regression model UTASTAR with DEA, in a manner that enables us to effectively incorporate the output of the former in the DEA efficiency assessments. In phase I we employ the ordinal regression method UTASTAR to assess value functions consistent with the user's preferences stated on a sample of DMUs. In phase II, we incorporate the assessed value model in a value-based DEA framework to assess the efficiency of the entire set of DMUs.

Acknowledgement

This research has been co-financed by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: THALES - Investing in knowledge society through the European Social Fund.

REFERENCES

[1] Almeida, P.N., & Dias, L.C. (2012). Value-based DEA models: application-driven developments. Journal of the Operational Research Society, 63, 16-27.

[2] Banker, R.D., Charnes, A., & Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science, 30, 1078-1092.

[3] Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European Journal of Operational Research, 2, 429-444.

[4] Charnes, A., Cooper, W. W., Wei, Q. L., & Huang, Z. M. (1989). Cone ratio data envelopment analysis and multi-objective programming. International Journal of Systems Science, 20, 1099-1118.

[5] Cook, W. D., Kazakov, A., Roll, Y., & Seiford, L. M. (1991). A data envelopment analysis approach to measuring efficiency: Case analysis of highway maintenance patrols. The Journal of Socio-Economics, 20, 83-103.

[6] Cook, W. D., & Zhu, J. (2009). Piecewise linear output measures in DEA. European Journal Operational Research, 197, 312-319.

[7] Despotis, D. K., Stamati, L. V., & Smirlis, Y. G. (2010). Data envelopment analysis with nonlinear virtual inputs and outputs. European Journal of Operational Research, 202, 604-613.

[8] Despotis, D. K., & Sotiros, D. (2013). Value-based Data Envelopment Analysis: A piece-wise linear programming approach. International Journal of Multicriteria Decision Making, to appear.

[9] Dyson, R. G., & Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. Journal of the Operational Research Society, 39, 563-576.

[10] Golany, B., & Roll, Y. (1994). Incorporating Standards in DEA. In Charnes, A. et al (Eds.), Data Envelopment Analysis: Theory, Methodology and Applications (393-422). Kluwer Academic Publishers.

[11] Gouveia, M.C., Dias, L.C., & Antunes, C.H. (2008). Additive DEA based on MCDA with imprecise information. Journal of the Operational Research Society, 59, 54-63.

[12] Jacquet-Lagreze E., & Siskos J. (1982). Assessing a set of additive utility functions for multicriteria decision-making, the UTA method. European Journal of Operational Research, 10, 151-164.

[13] Podinovski, V.V. (2004). Production trade-offs and weight restrictions in data envelopment analysis. Journal of the Operational Research Society, 55, 1311-1322.

[14] Siskos, J., & Yannacopoulos, D. (1985). An Ordinal Regression Method for Building Additive Value Functions. Investigacao Operacional, 5.

[15] Thanassoulis, E., & Allen, R. (1998). Simulating weights restrictions in data envelopment analysis by means of unobserved DMUs. Management Science, 44, 586-594.