“A COST MALMQUIST PRODUCTIVITY INDEX CAPTURING GROUP PERFORMANCE ”

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THE TRADITIONAL MALMQUIST INDEX AND ITS COST VARIANT

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$$IM = \left[\frac{D^t\left(y^{t+1}, x^{t+1}\right)}{D^t\left(y^t, x^t\right)} \frac{D^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D^{t+1}\left(y^t, x^t\right)} \right]^{1/2}$$

$$IM = \left[\frac{w^t x^{t+1} / C^t\left(y^{t+1}, w^t\right)}{w^t x^t / C^t\left(y^t, w^t\right)} \frac{w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1}, w^{t+1}\right)}{w^{t+1} x^t / C^{t+1}\left(y^t, w^{t+1}\right)} \right]^{1/2}$$

$$w^t x^t = \sum_{n=1}^{N} w_n^t x_n^t$$

$$C^t(y^t, w^t) = \min \sum_{i=1}^{m} w_{ij_0}^t x_i$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij}^t \le x_i \quad ,i = 1,\ldots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rj_0}^t, r = 1,\ldots,s,$$

$$\lambda_j \ge 0 \; ,x_i \ge 0, \quad j = 1,\ldots,n; i = 1,\ldots,m$$

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GROUP PERFORMANCE MALMQUIST INDEX

$$I^{AB} = \left[\frac{\left(\prod_{j=1}^{\delta_A} D^A(X_j^A, Y_j^A) \right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^A(X_j^B, Y_j^B) \right)^{1/\delta_B}} \times \frac{\left(\prod_{j=1}^{\delta_A} D^B(X_j^A, Y_j^A) \right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} D^B(X_j^B, Y_j^B) \right)^{1/\delta_B}} \right]^{1/2}$$

I^{AB} is an overall measure for the comparison of performance between two groups of DMUs *A* and B, associated with different programs or environmental conditions.

A COST INDEX FOR COMPARING GROUPS OF DMUS

Define the cost efficiency for DMU j of Group A as

$$CE^{A}(X_j^A, Y_j^A, W^A) = \frac{C^A(Y_j^A, W^A)}{W^A X_j^A}$$

Using the Group A cost frontier as reference level the Index for comparing DMUs in Group A to those in Group B is

$$CI^{A} = \left[\frac{\left(\prod_{j=1}^{\delta_A} W^A X_j^A \big/ C^A(Y_j^A, W_j^A) \right)^{\frac{1}{\delta_A}}}{\left(\prod_{j=1}^{\delta_B} W^A X_j^B \big/ C^A(Y_j^B, W_j^A) \right)^{\frac{1}{\delta_B}}} \right] = \left[\frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B} \right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{A} \right)^{\frac{1}{\delta_A}}} \right]$$

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A COST MALMQUIST INDEX FOR COMPARING GROUPS

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An index similar to CI^A can be defined using the Group B cost frontier as reference level, yielding CI^B

The geometric mean of CI^A and CI^B yields a Group Cost Malmquist Index

$$CI^{AB} = \left(CI^A \times CI^B\right)^{0.5} = \left[\frac{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A}\right)^{\frac{1}{\delta_A}}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B}\right)^{\frac{1}{\delta_B}}}{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{\frac{1}{\delta_A}}} \right]^{0.5}$$

DECOMPOSITION OF THE GROUP COST MALMQUIST INDEX

$$CI^{AB} = OECG^{AB} \times CTCG^{AB}$$

$$OECG^{AB} = \frac{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{1/\delta_A}}$$

= Group overall cost efficiency change

$$CTCG^{AB} = \left[\frac{\left(\prod_{j=1}^{\delta_A} CE_j^A\right)^{1/\delta_A}}{\left(\prod_{j=1}^{\delta_B} CE_j^B\right)^{1/\delta_B}} \times \frac{\left(\prod_{j=1}^{\delta_B} CE_j^{A,B}\right)^{1/\delta_B}}{\left(\prod_{j=1}^{\delta_A} CE_j^{B,A}\right)^{1/\delta_A}}\right]^{1/2}$$

= Group cost technical change

TWO-LEVEL DECOMPOSITION OF THE GROUP COST MALMQUIST INDEX

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$$CI^{AB} = \text{overall efficiency change group}\left(OECG^{AB}\right) \times \text{cost technical change group}\left(CTCG^{AB}\right)$$

$$= \text{technical efficiency change group}\left(TECG^{AB}\right) \times \text{allocative efficiency change group}\left(AECG^{AB}\right) \times$$

$$\text{technical change group}\left(TCG^{AB}\right) \times \text{price technical-effect group}\left(PEG^{AB}\right)$$

$$= I^{AB} \times \text{allocative efficiency change group}\left(AECG^{AB}\right) \times \text{price technical-effect group}\left(PEG^{AB}\right)$$

Second stage decomposition

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- Group overall efficiency change can be decomposed into *technical efficiency change – group* ($TECG^{AB}$) and *allocative efficiency change-group* ($AECG^{AB}$)

- The **$CTCG^{AB}$ (Group Cost Technology Change)** component of CI^{AB} can be decomposed into a *technical change - Group* (TCG^{AB}) and a *price-technical effect - Group* (PEG^{AB}) component.

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Conclusion

We have addressed the situation where we have Groups of DMUs operating under different contexts which can include different input prices

- We have defined an index which reflects the overall performance in aggregate cost terms of one Group of DMUs relative to another.

- The index can be multiplicatively decomposed in a first stage into two indices, one reflecting the relative bunching of DMUs in each Group and the other the mean distance between the cost frontiers of groups *A* and *B*, akin to the boundary shift in the traditional Malmquist index.

- The two first stage decomposition indices can themselves be decomposed into two indices each.

THANK YOU